

# A BAYESIAN APPROACH TO INTERNAL MODELS

MÁTÉ LENGYEL

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Department of Engineering  
University of Cambridge



Department of Cognitive Science  
Central European University



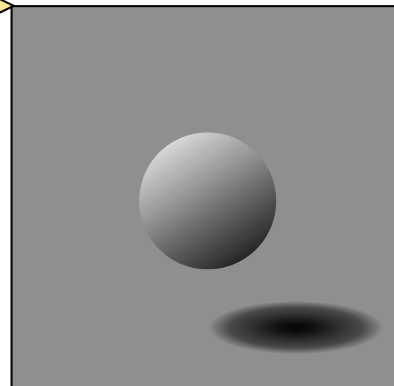
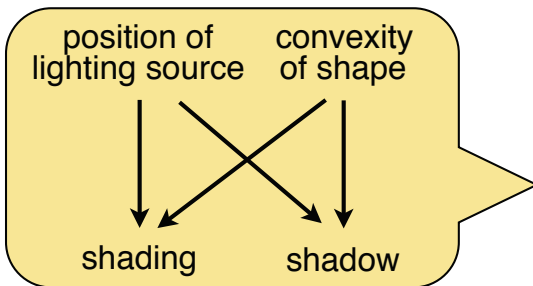
# INTERNAL MODELS

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**internal model:**  
a mental representation of  
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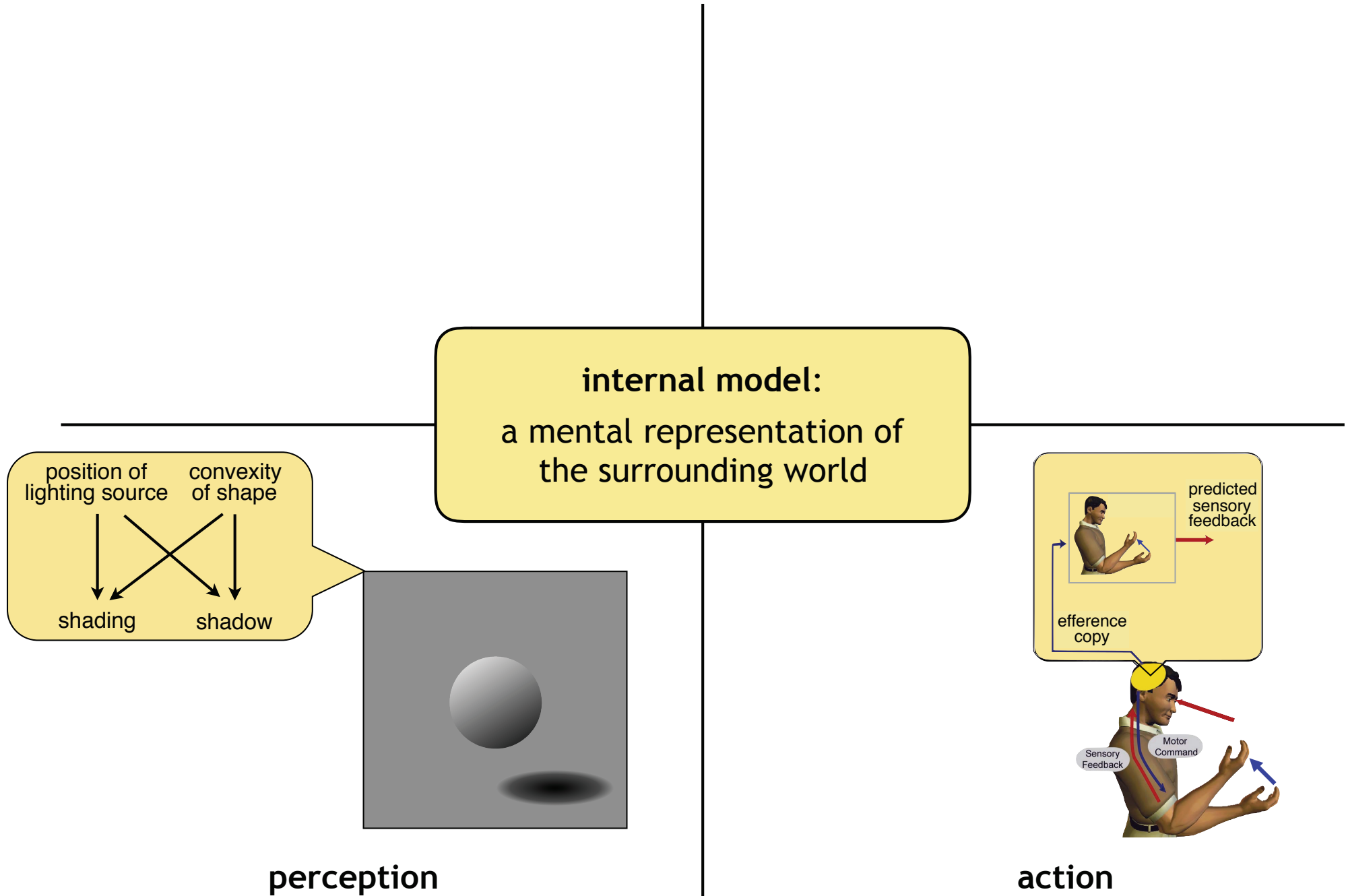
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**perception**

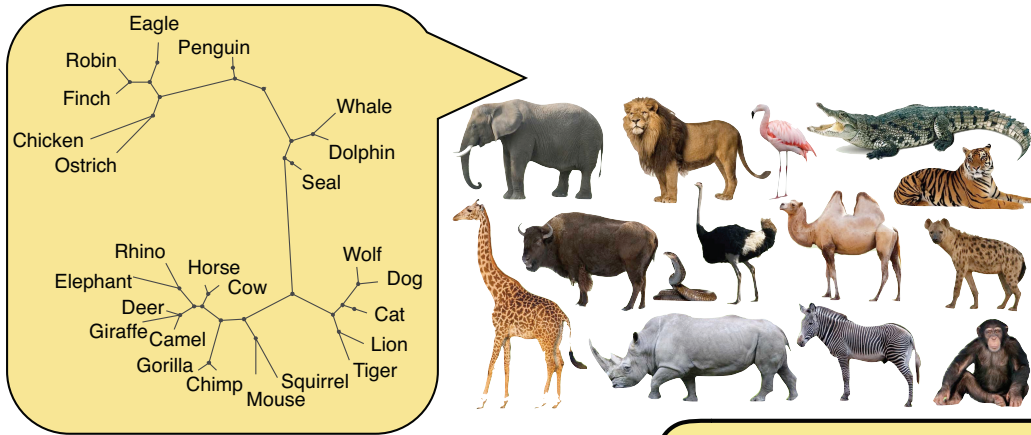


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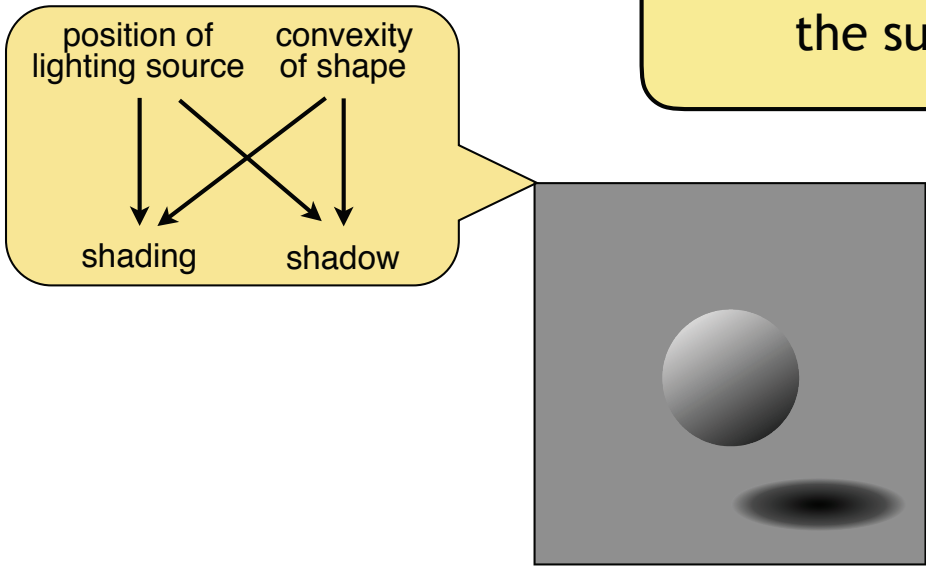


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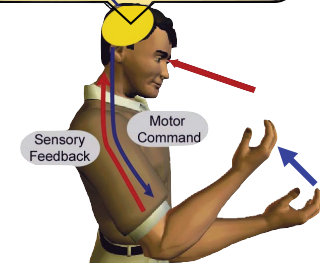
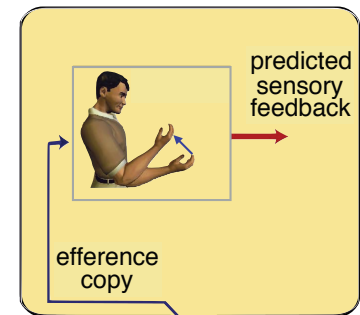
## semantic knowledge



**internal model:**  
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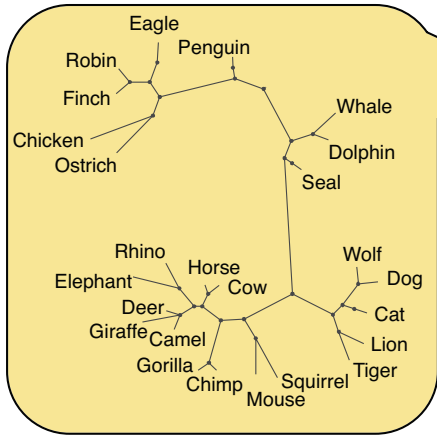
## perception



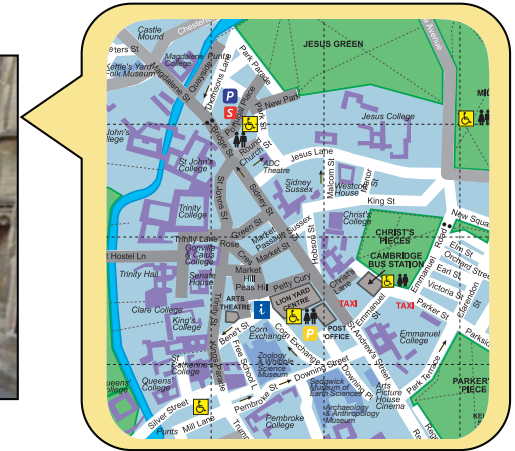
## action

# INTERNAL MODELS

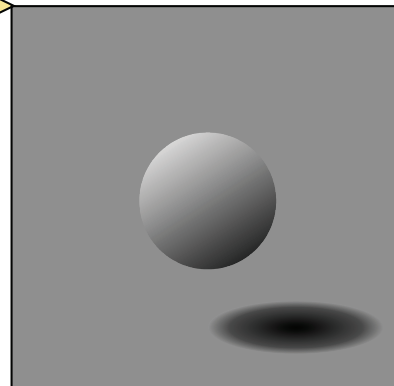
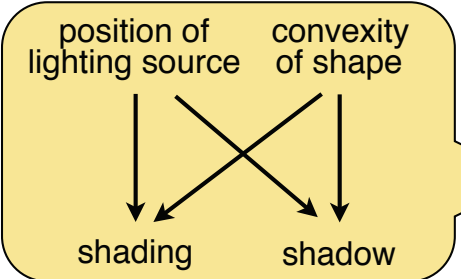
semantic knowledge



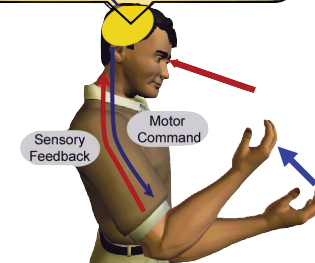
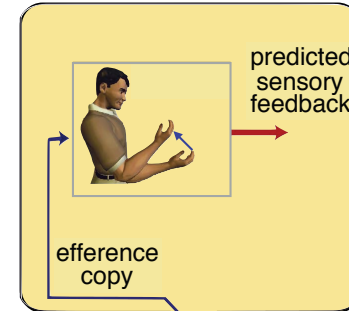
navigation



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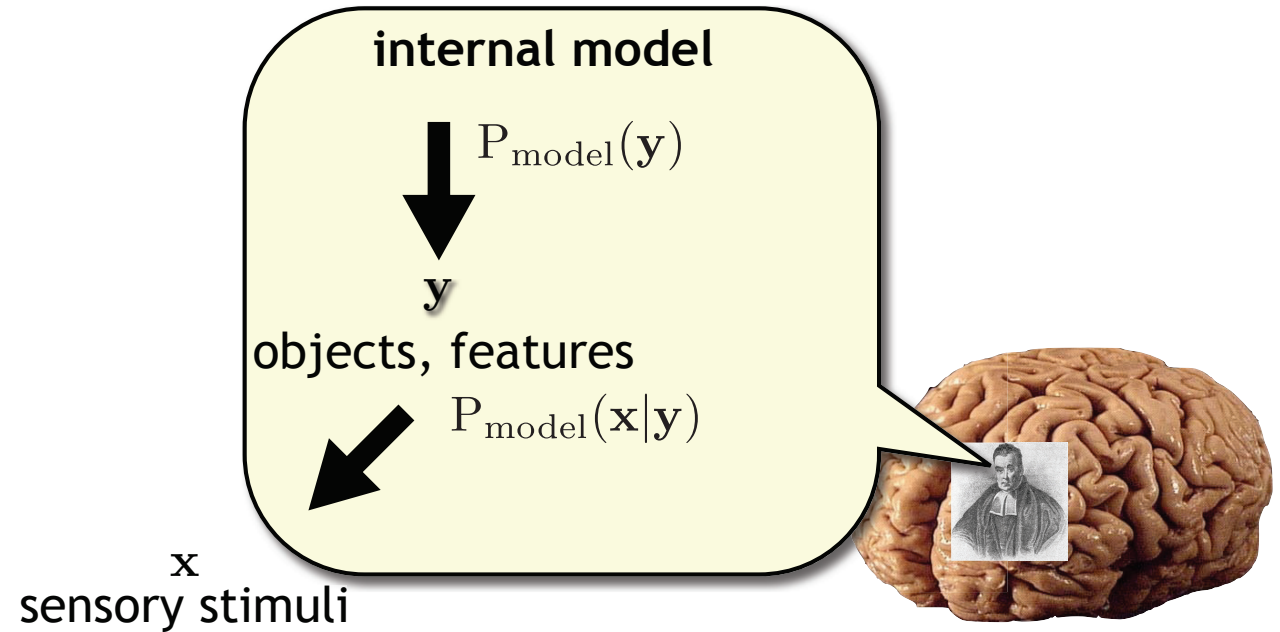
perception



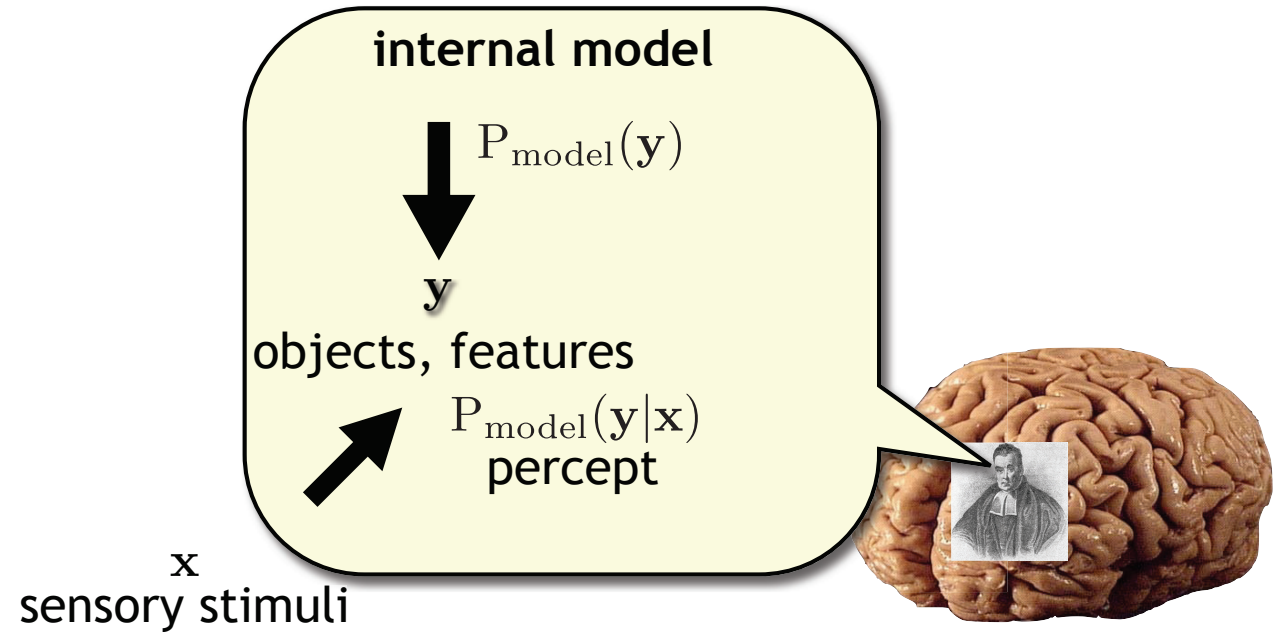
action

# PROBABILISTIC INTERNAL MODELS

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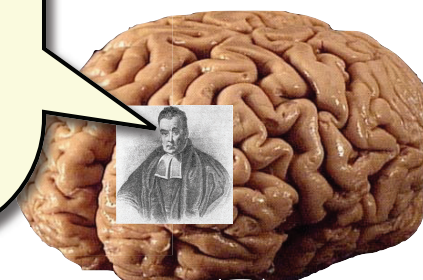
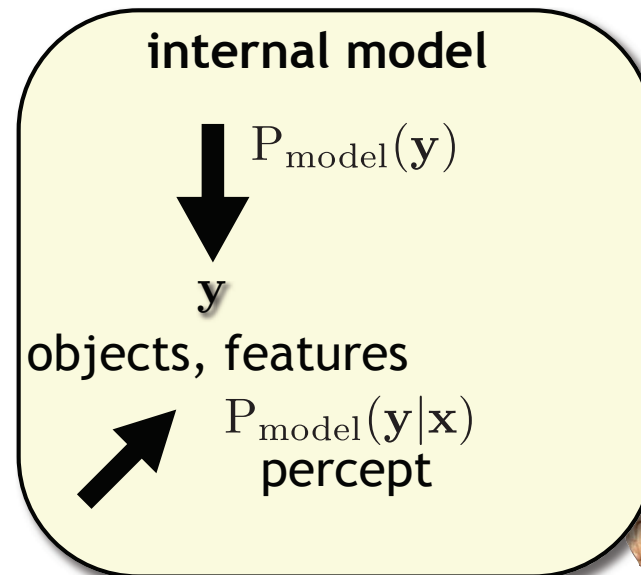
# PROBABILISTIC INTERNAL MODELS



$P_{\text{true}}(\mathbf{y})$   
↓  
 $\mathbf{y}$   
objects, features

$P_{\text{true}}(\mathbf{x}|\mathbf{y})$   
↘

$\mathbf{x}$   
sensory stimuli



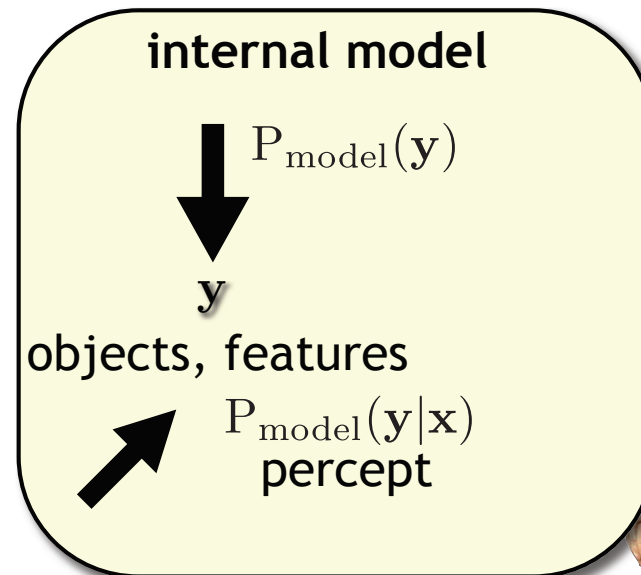
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## part 1. ADAPTATION TO ENVIRONMENT



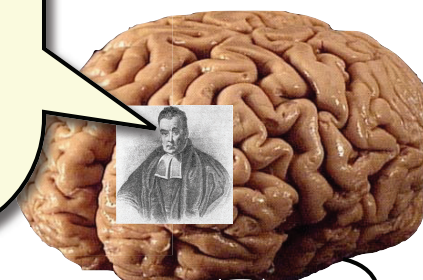
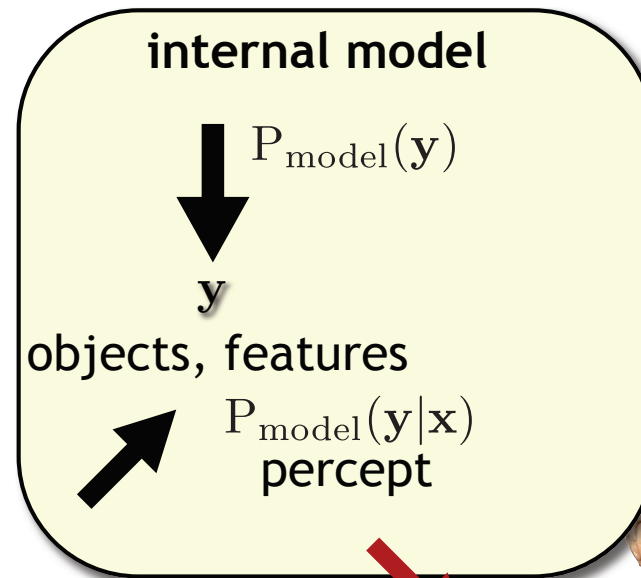
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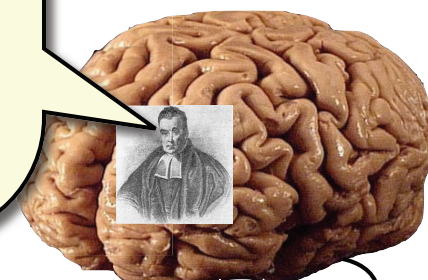
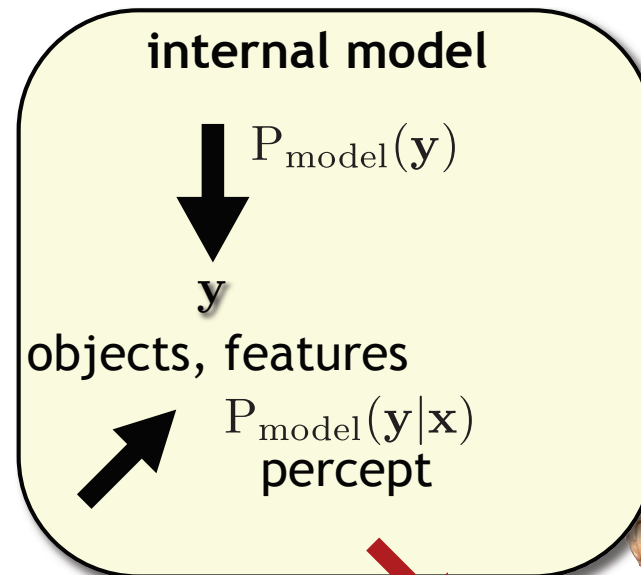
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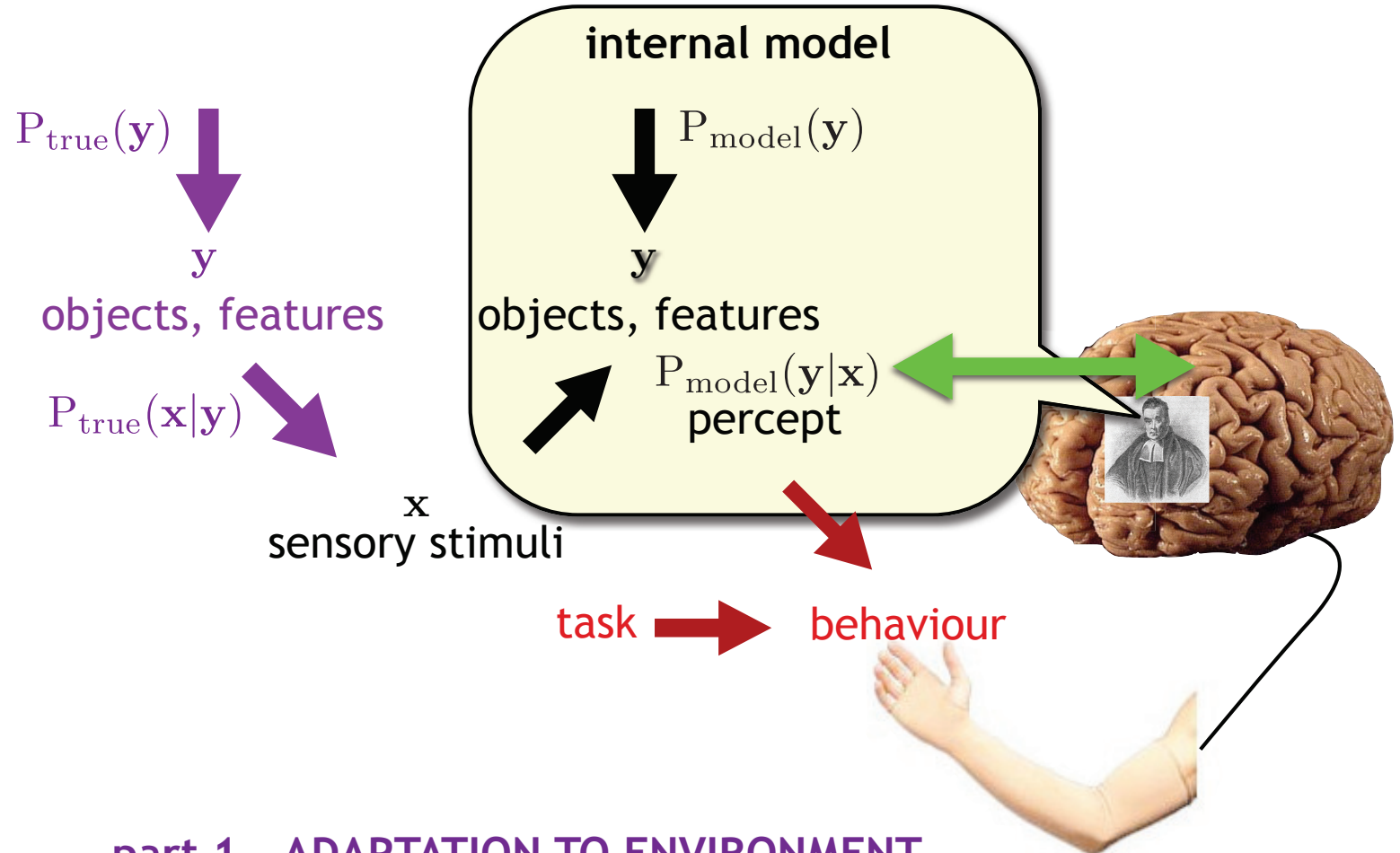
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sensory stimuli



part 1. ADAPTATION TO ENVIRONMENT

part 2. TASK-INDEPENDENCE

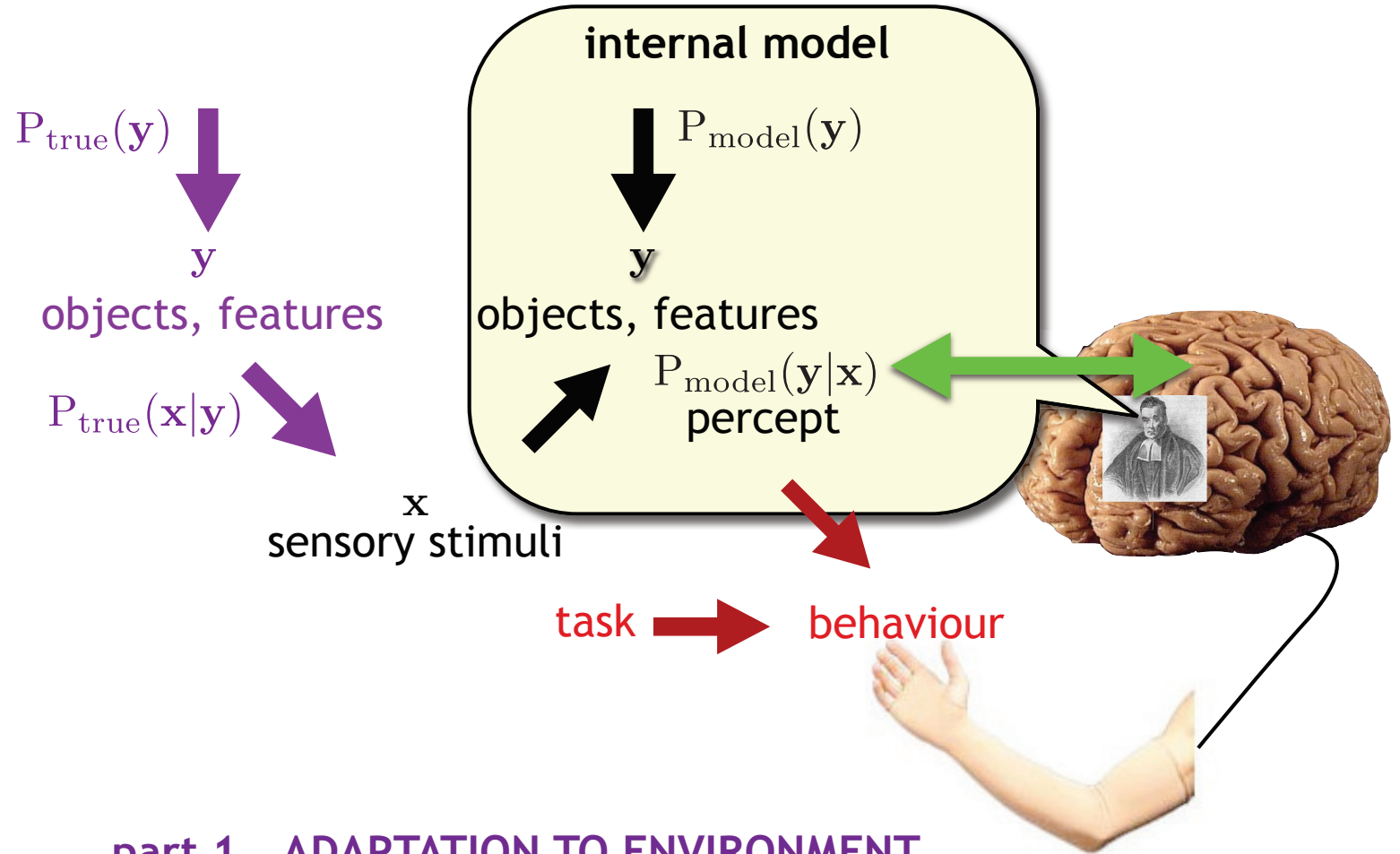
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part 2. TASK-INDEPENDENCE

part 3. NEURAL SUBSTRATES

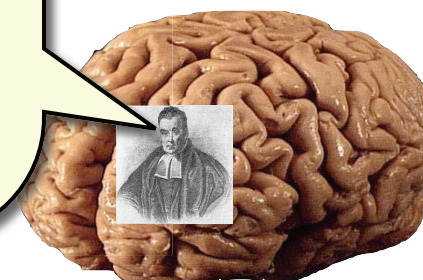
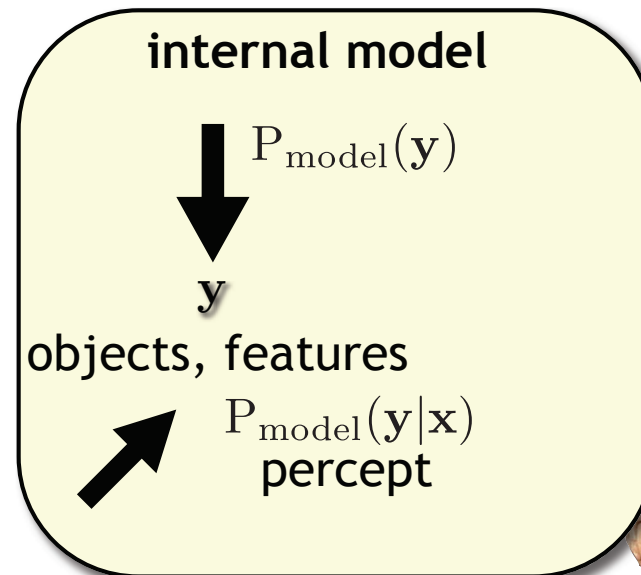
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# CHUNK LEARNING: HIERARCHICAL PAIR-WISE ASSOCIATIVE?

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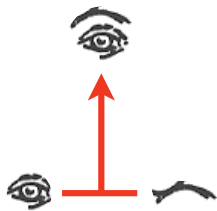




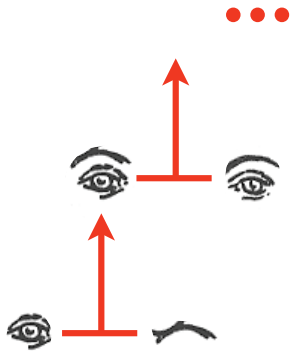
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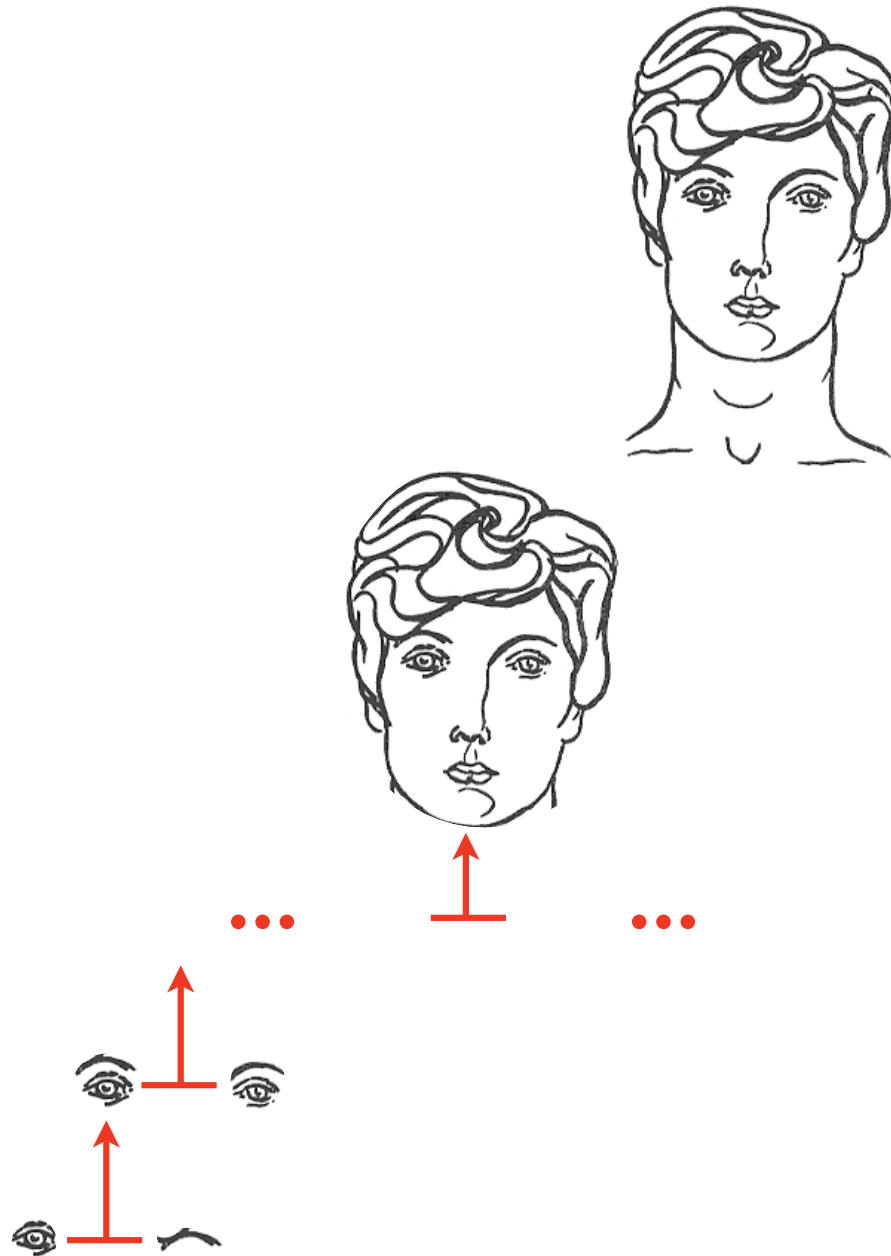
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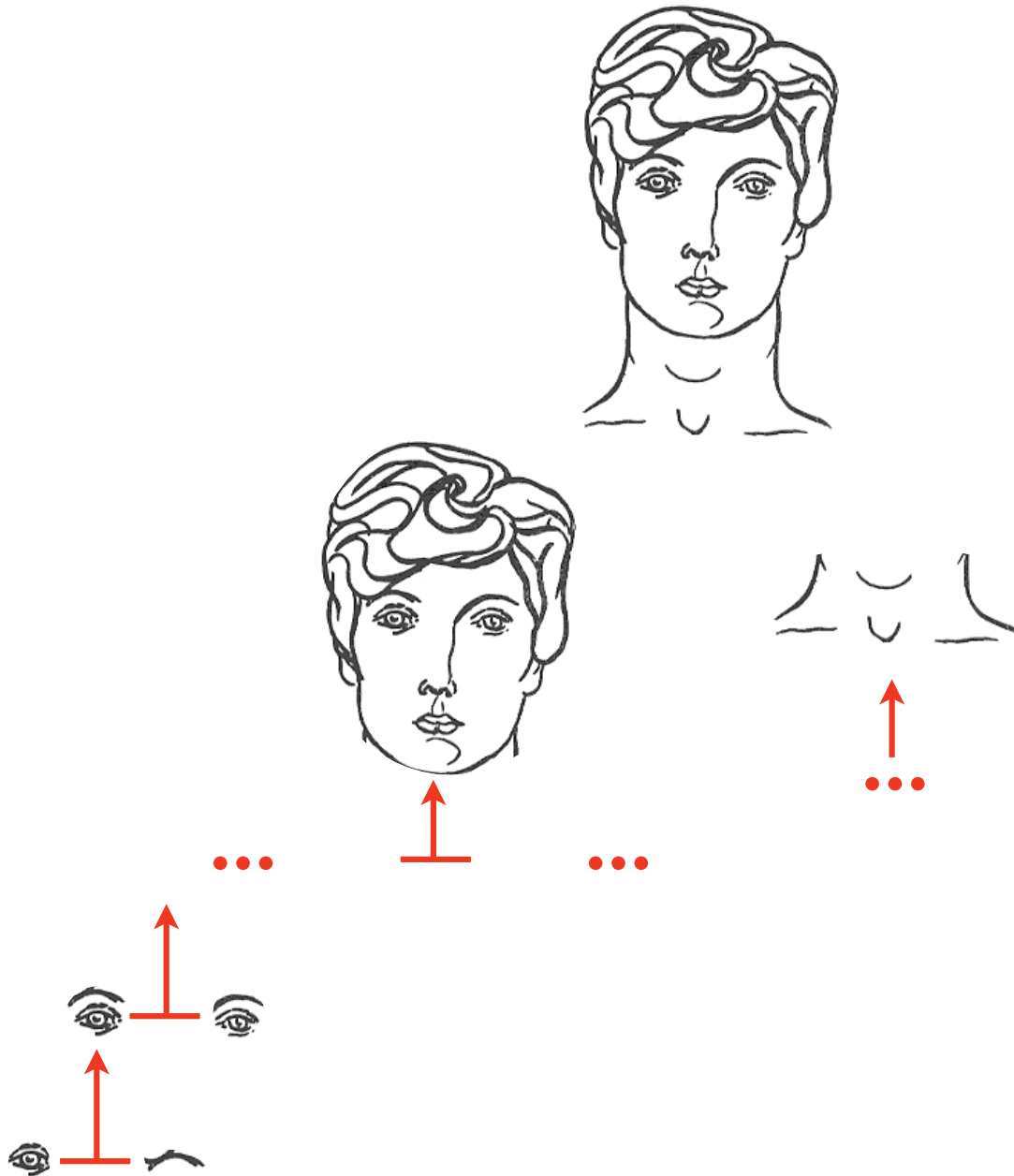
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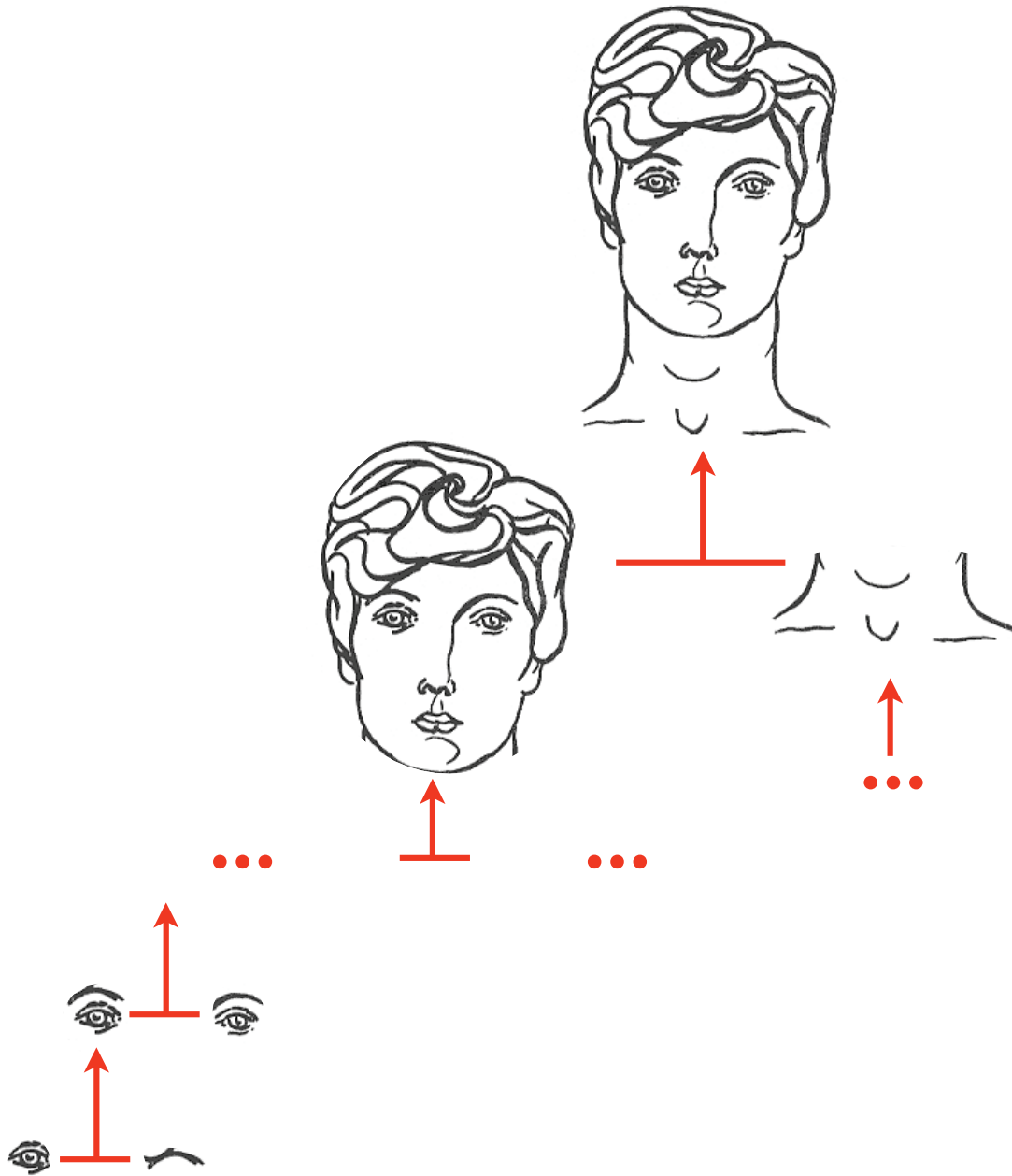
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storing pair-wise associations → learning about 2<sup>nd</sup> order statistics

**but the environment is much richer statistically!**

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natural image



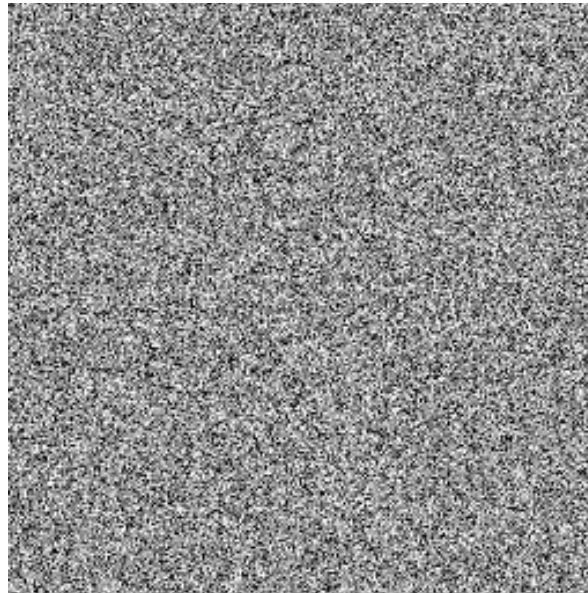
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predicted image  
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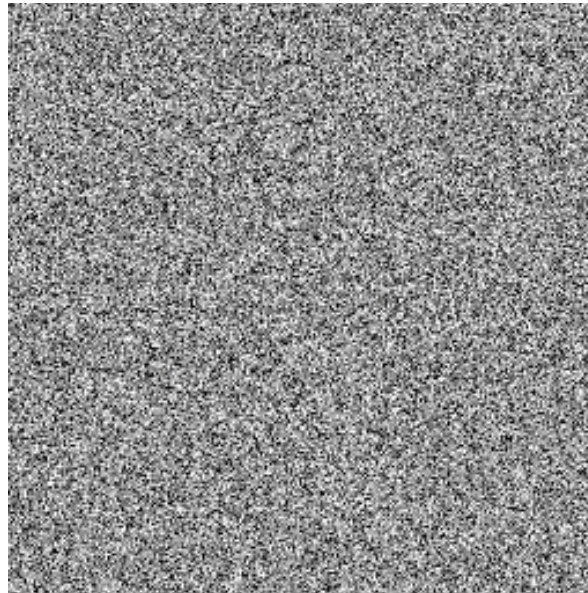
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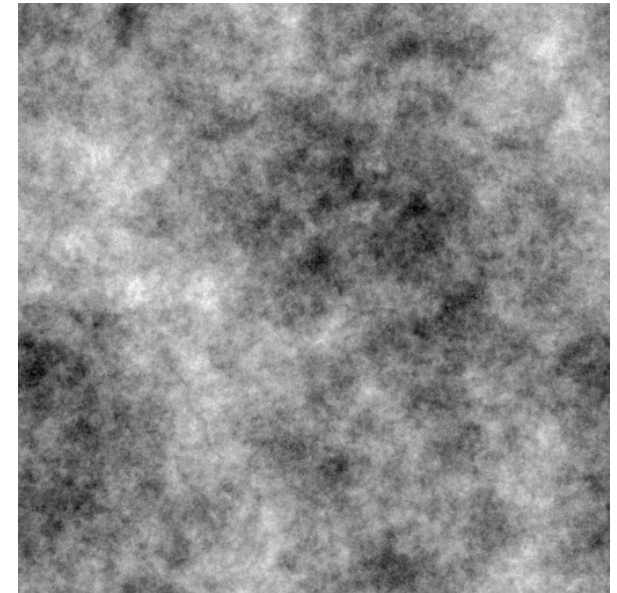
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predicted image  
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predicted image  
from 2<sup>nd</sup> order statistics

Bruno Olshausen

# THE PROBLEM WITH LEARNING HIGHER ORDER STATISTICS

- ▶ there are too many of them

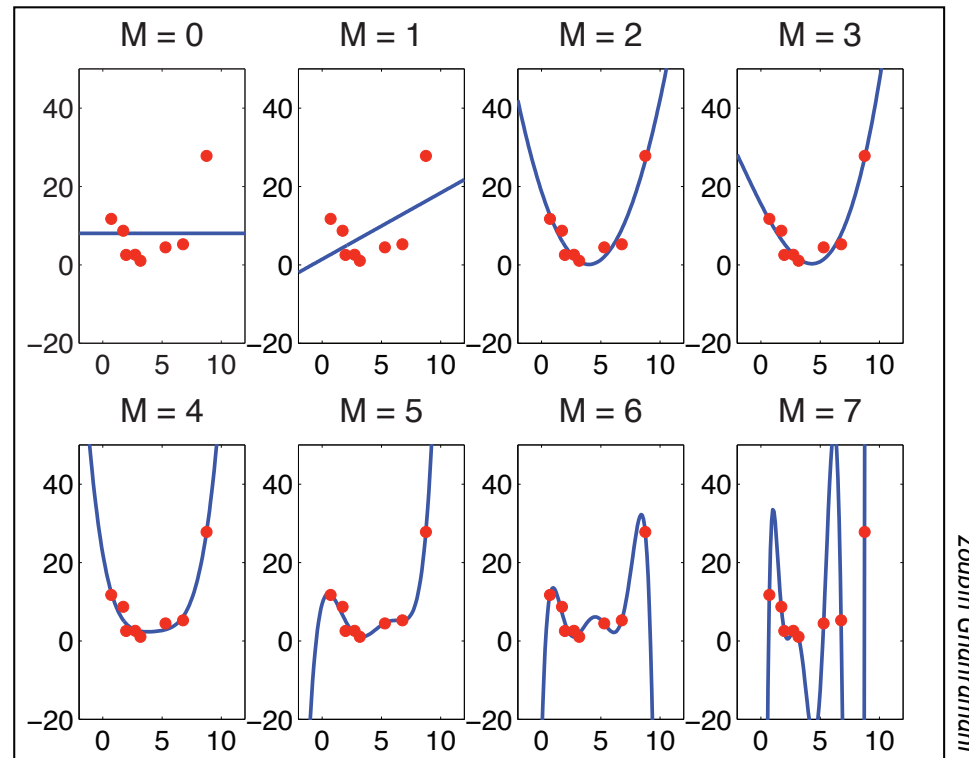
$N$ items	order of statistics	number of statistics
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- ▶ danger of overfitting





# BAYESIAN MODEL SELECTION

a model defines a probability distribution  
over data sets

$$P[\mathcal{D}|\mathcal{M}]$$

data  
coming from  
the environment

model  
stored in memory



Rev. Bayes

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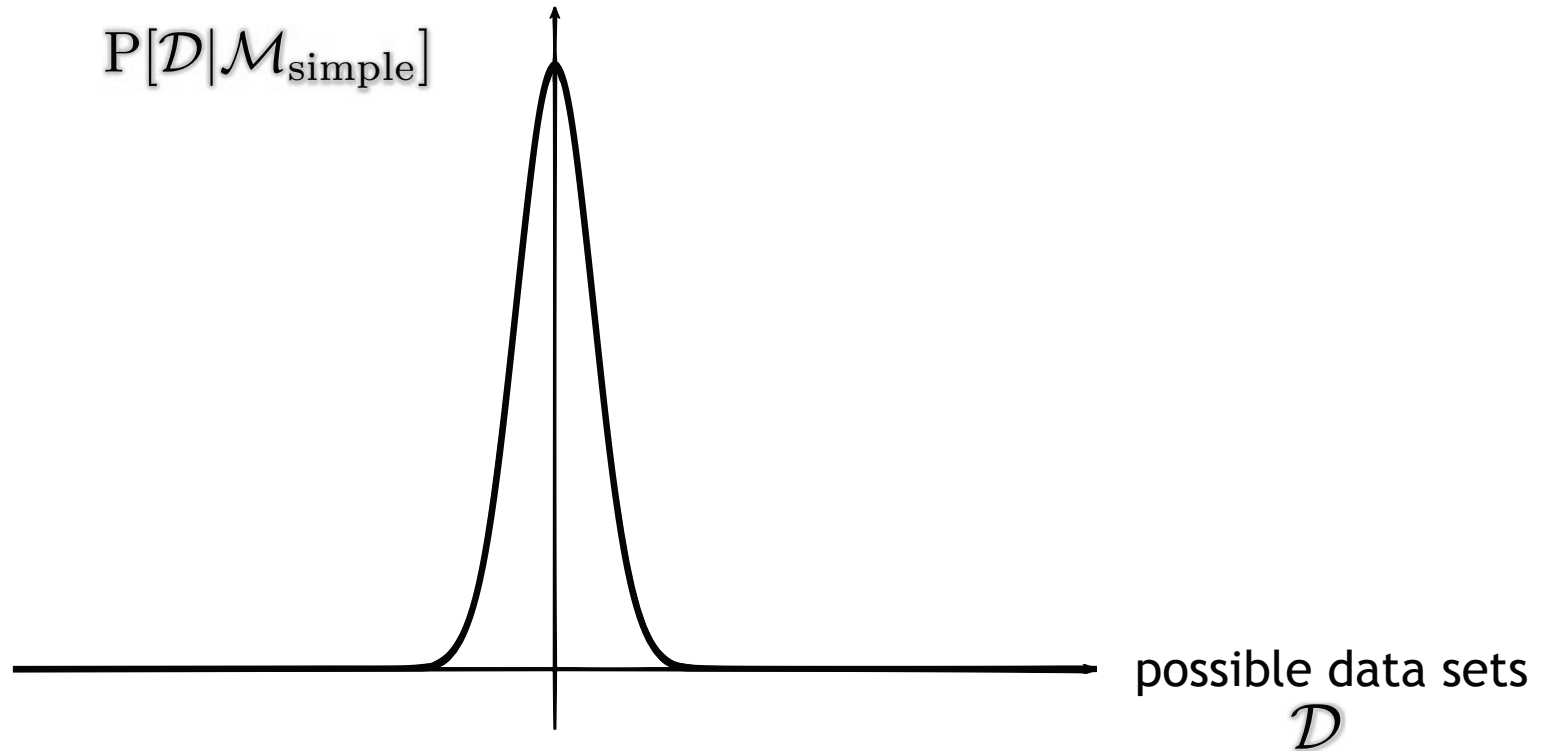
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Rev. Bayes

$$P[\mathcal{D}|\mathcal{M}_{\text{simple}}]$$



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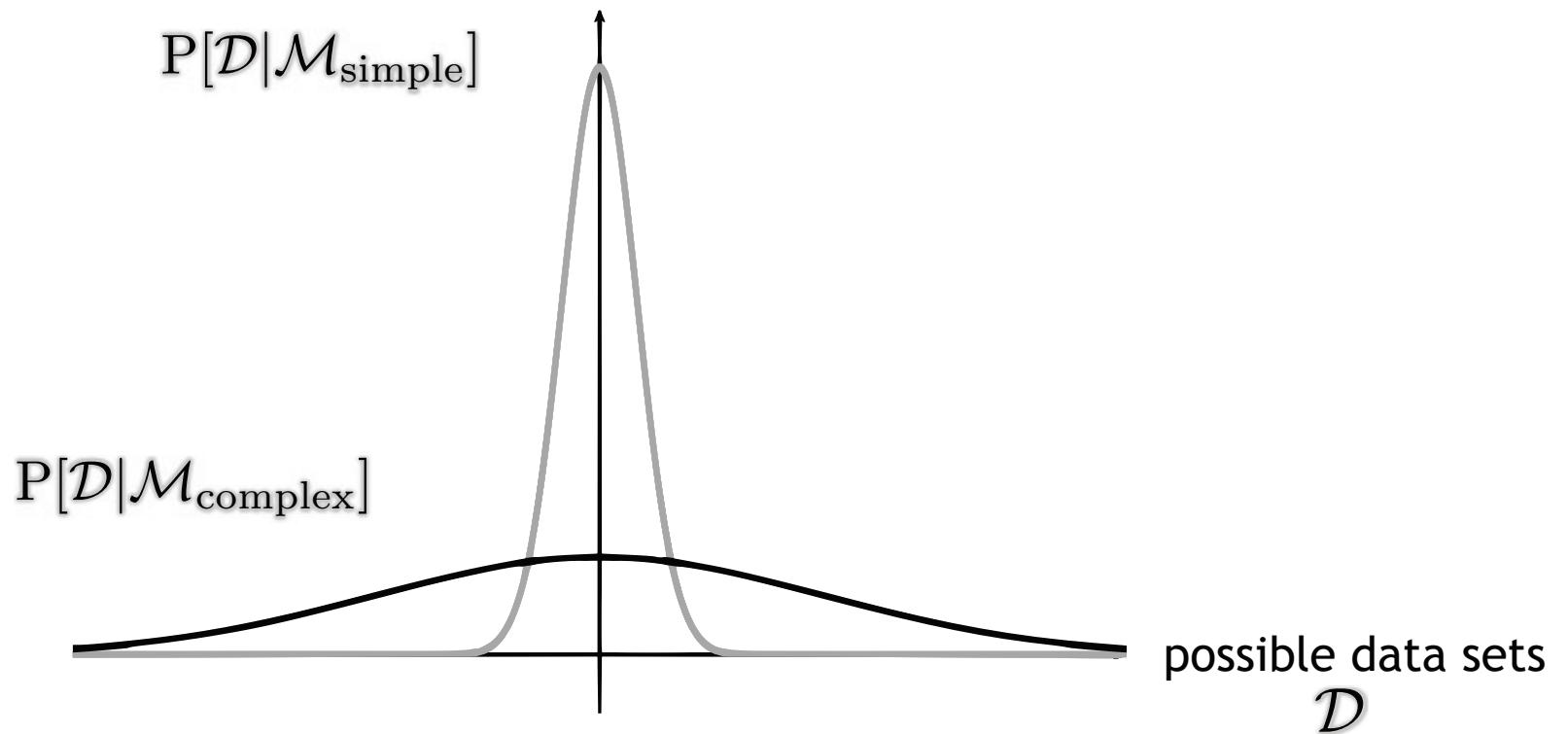
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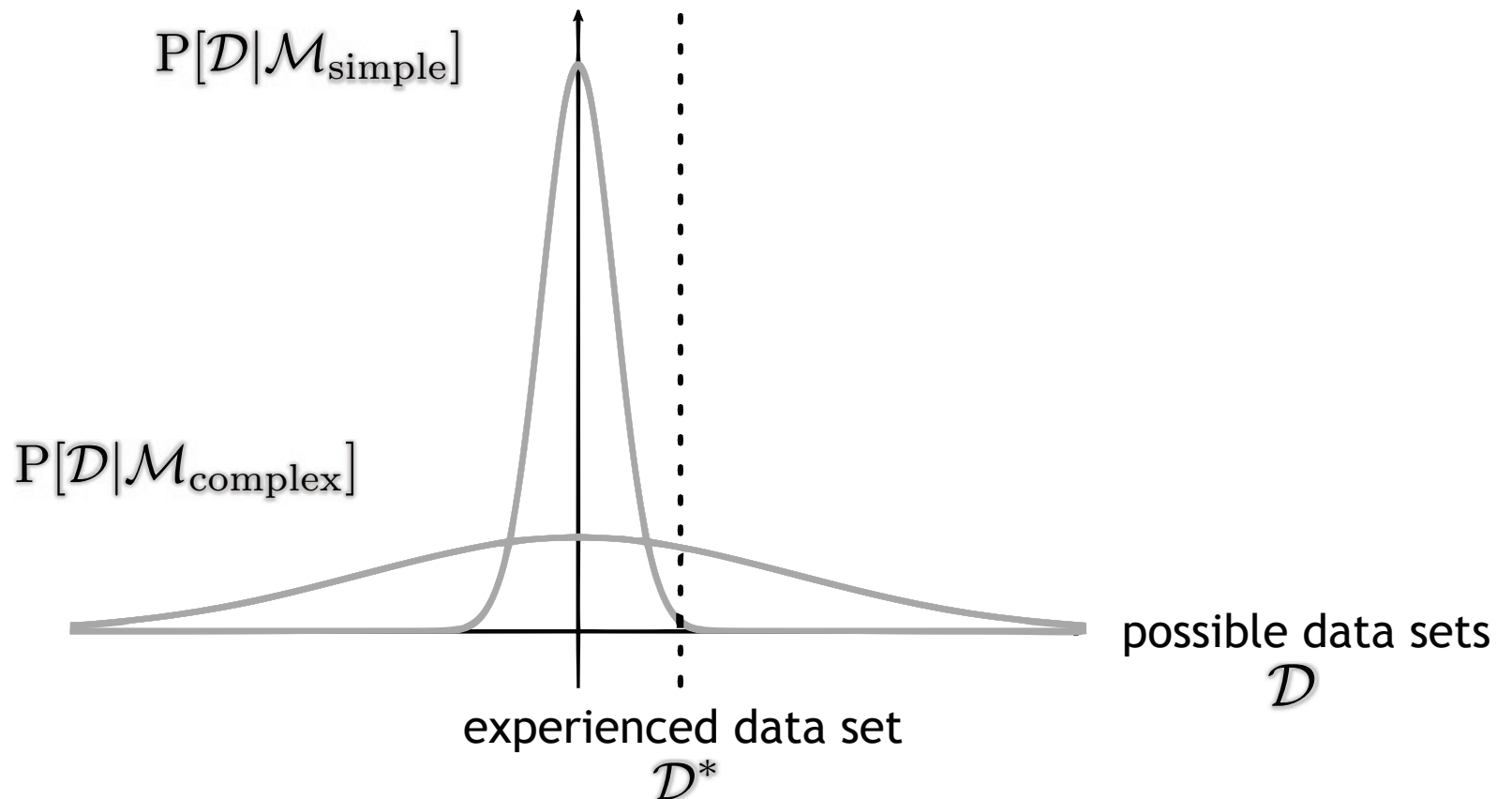
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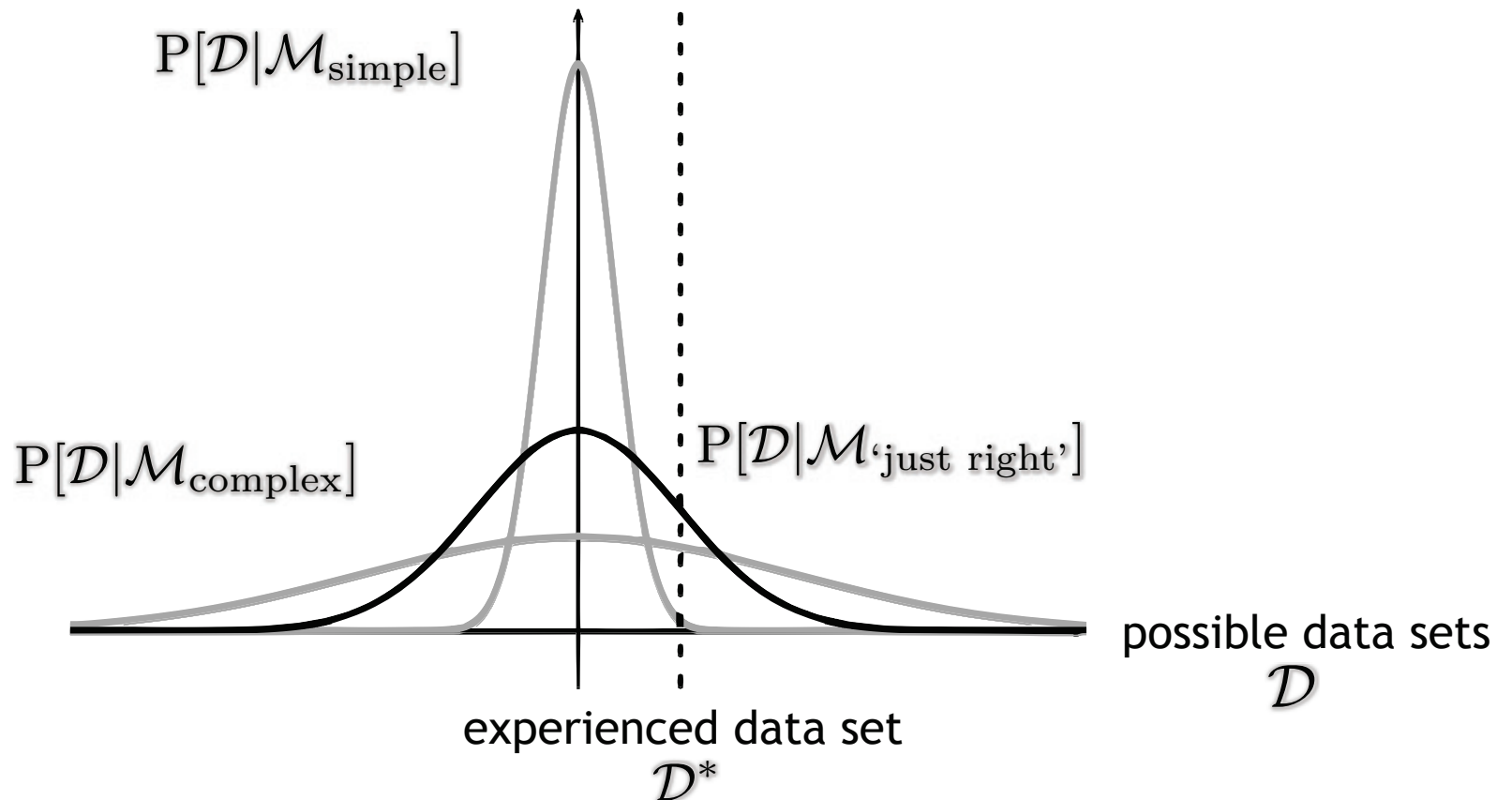
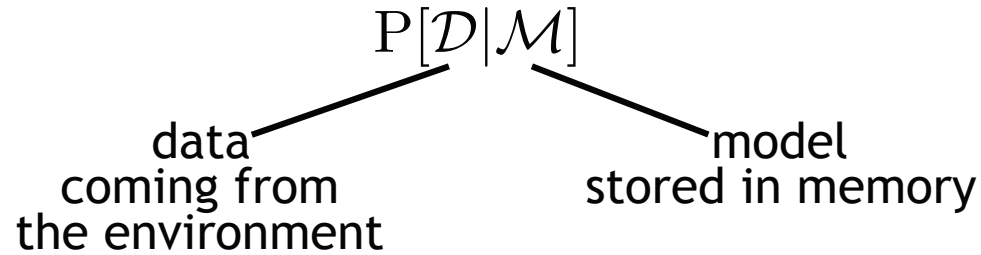


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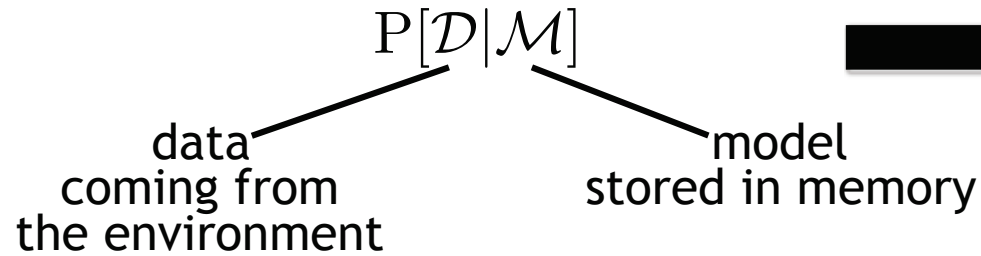


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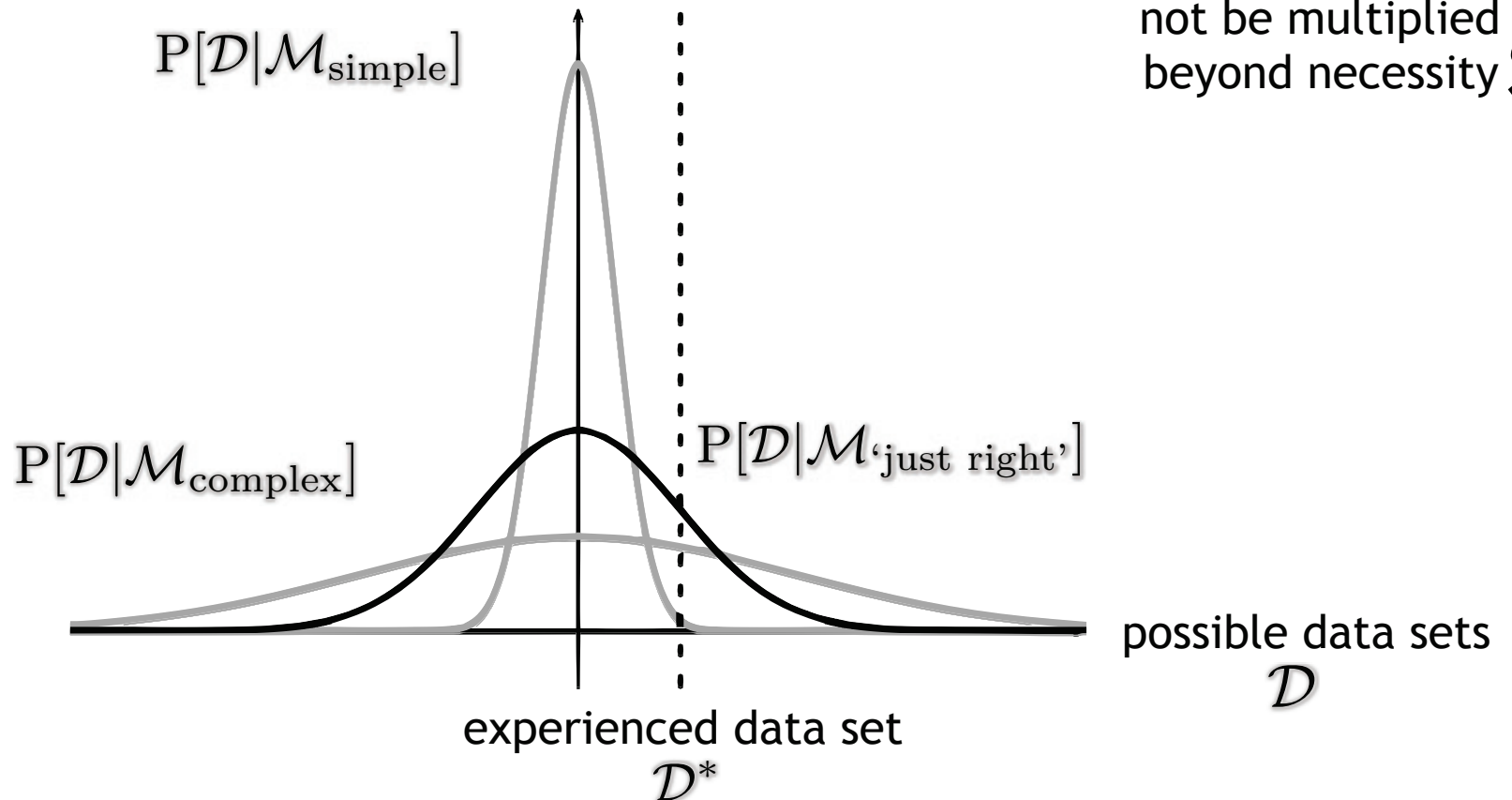
Rev. Bayes

a model defines a probability distribution over data sets



Occam's razor

‘ entities should not be multiplied beyond necessity ’



# VISUAL PATTERN LEARNING



Gergő  
Orbán



József  
Fiser



Richard  
Aslin

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**exposure**  
‘pay attention’

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‘which one looks more familiar?’

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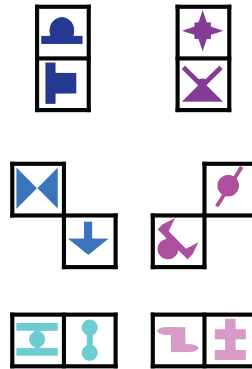
exposure

‘pay attention’

inventory

test

‘which one looks more familiar?’





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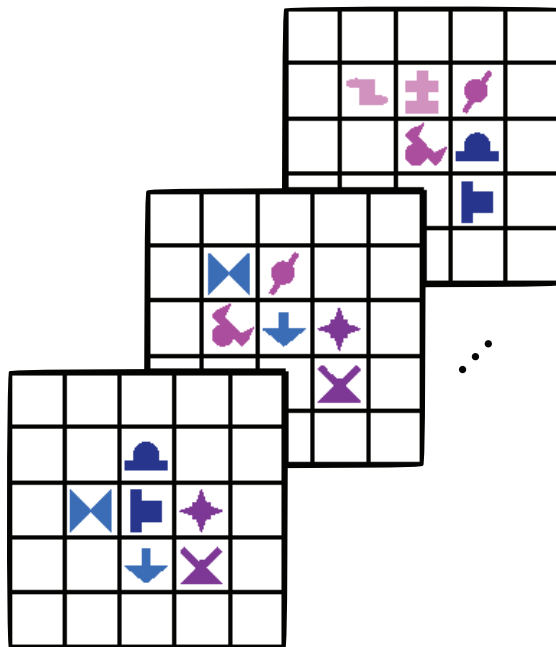
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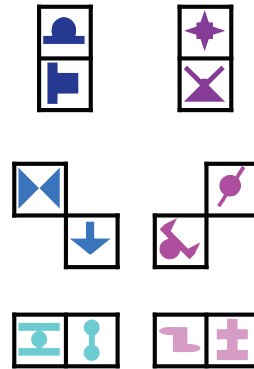
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Gergő Orbán



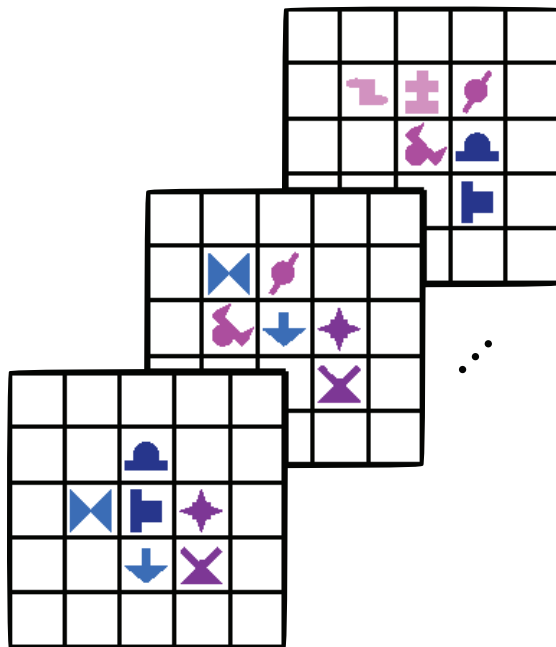
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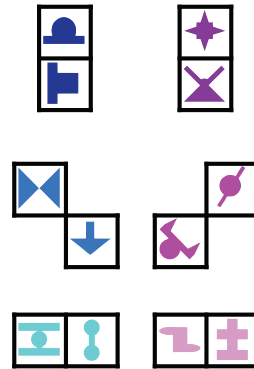
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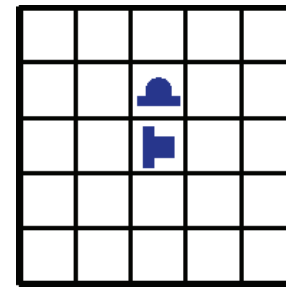


inventory

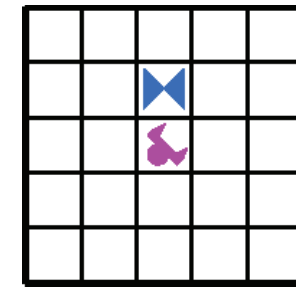


test

‘which one looks more familiar?’



VS.

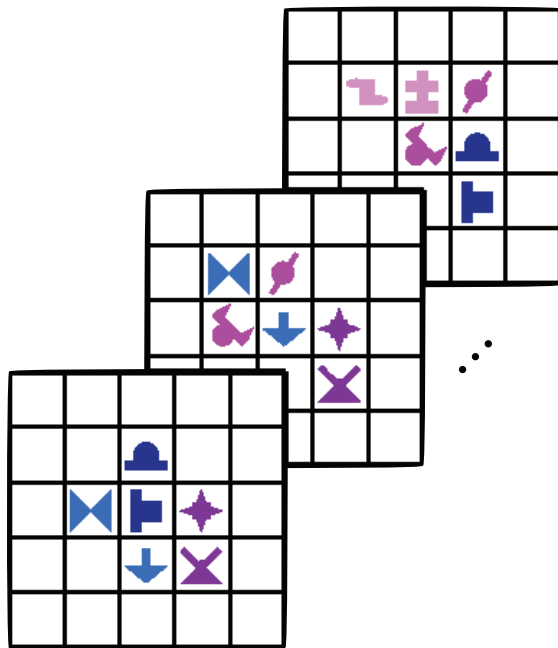


*Orbán & al, PNAS 2008*

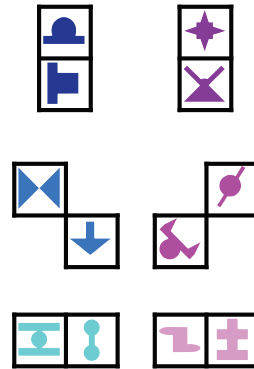
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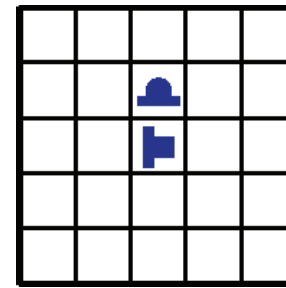


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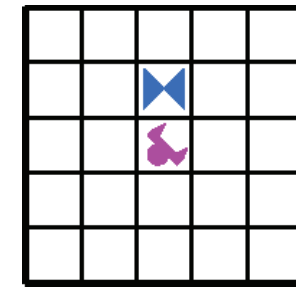


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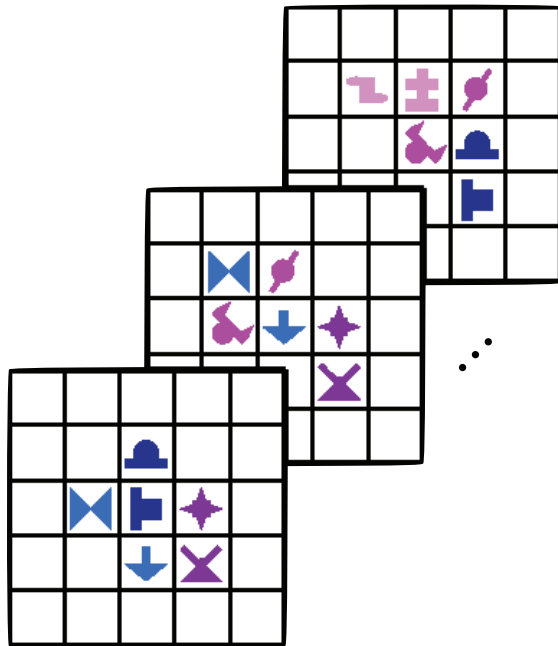
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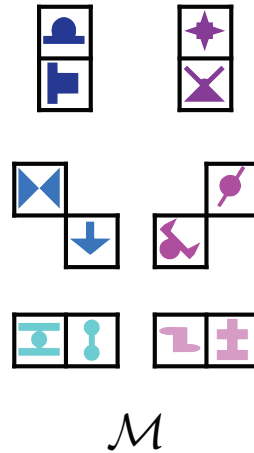
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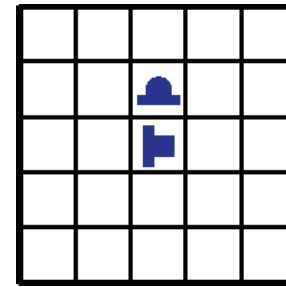


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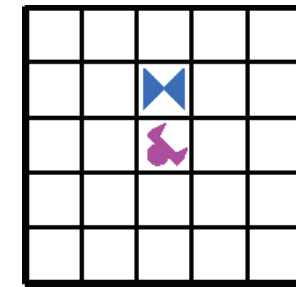


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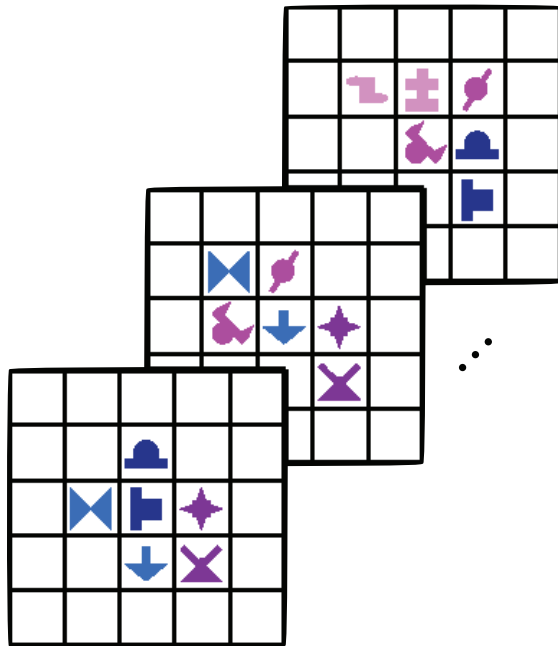
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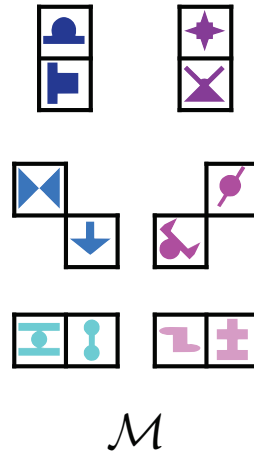
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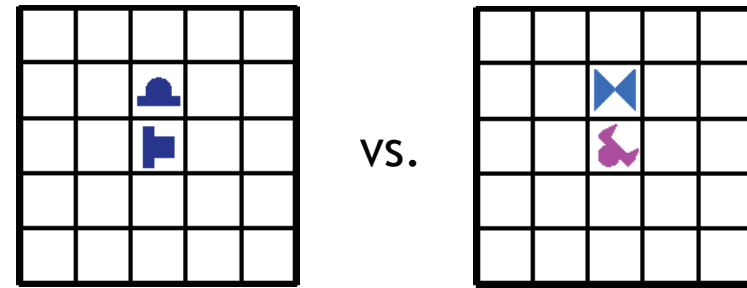


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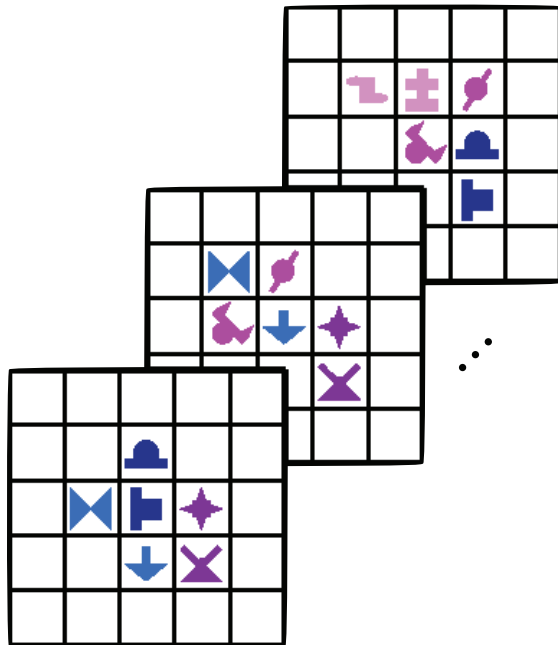
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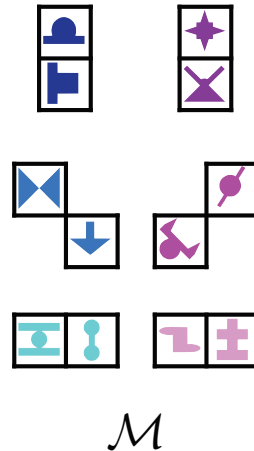
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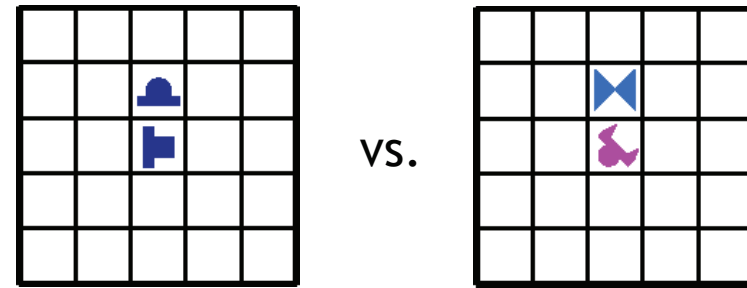


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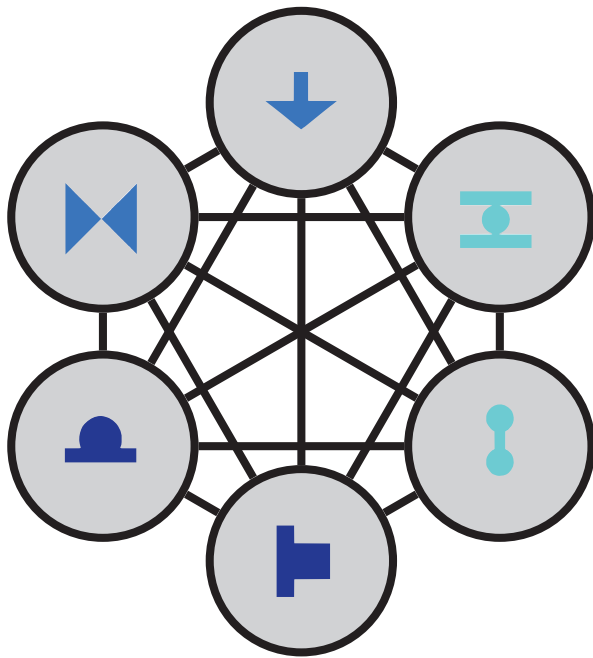
how do humans learn a statistical model of their environment?

- associative learning (fitting 2<sup>nd</sup> order max-entropy model)
- Bayesian model selection (inferring hidden causal structure)

# ALTERNATIVE THEORIES

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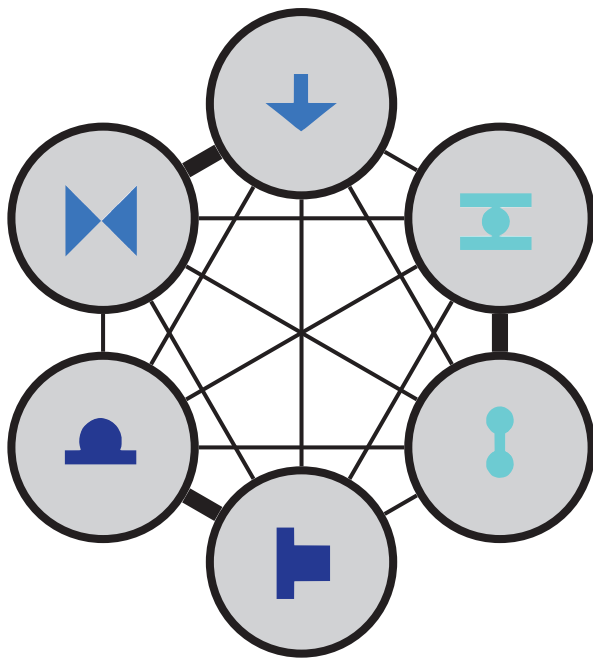
associative learning





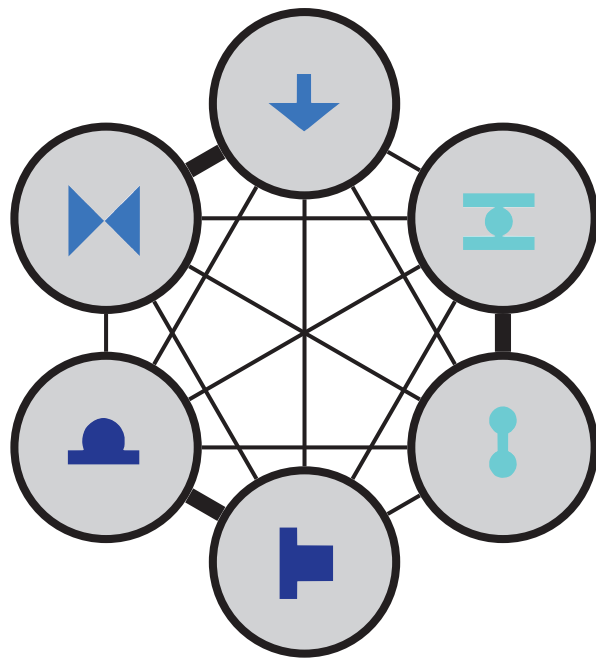
# ALTERNATIVE THEORIES

associative learning

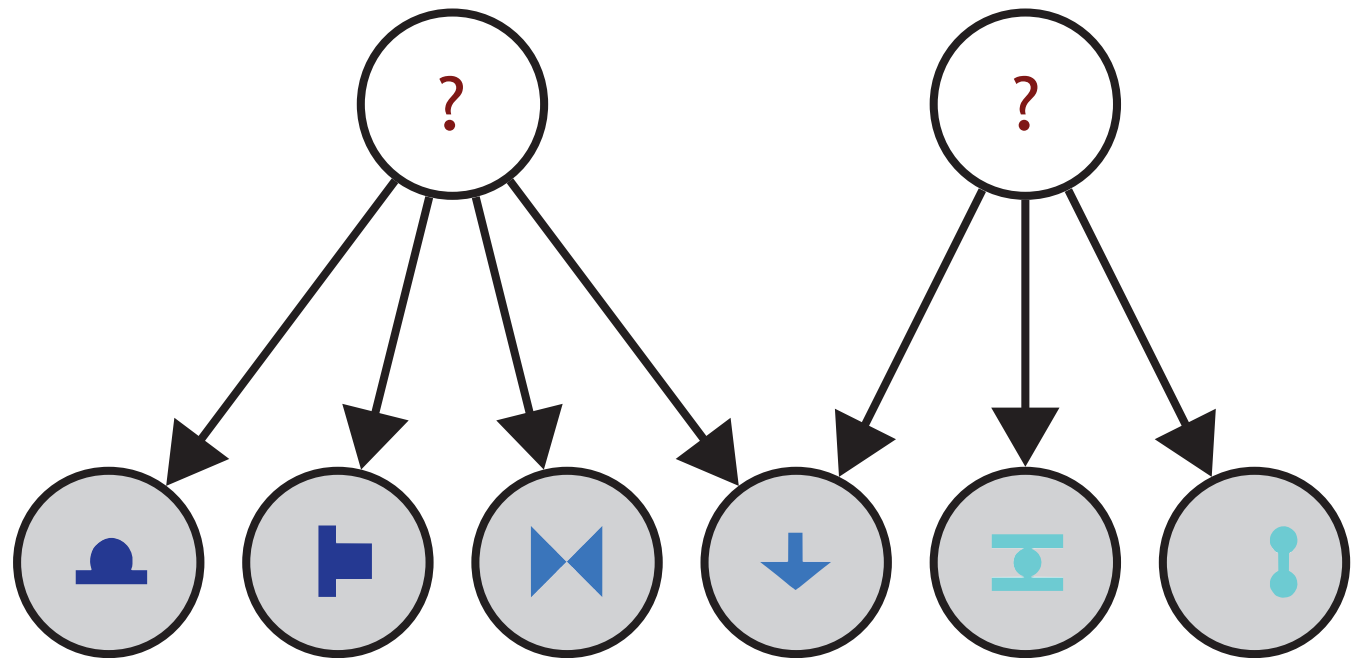


# ALTERNATIVE THEORIES

associative learning

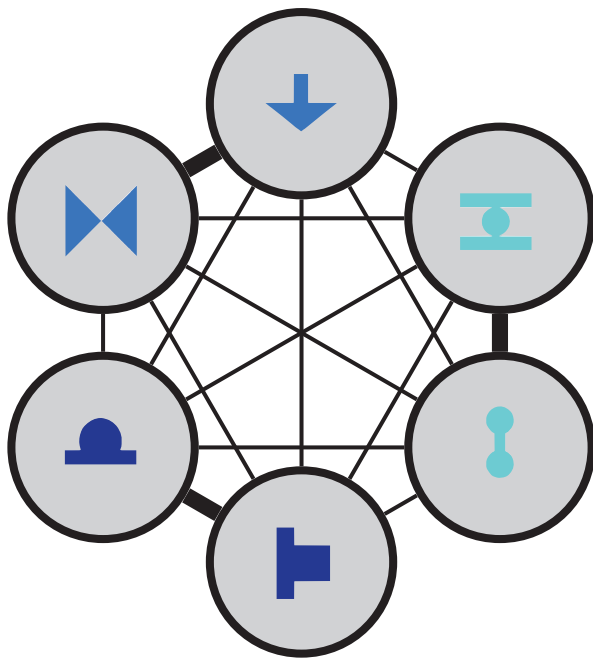


Bayesian learning

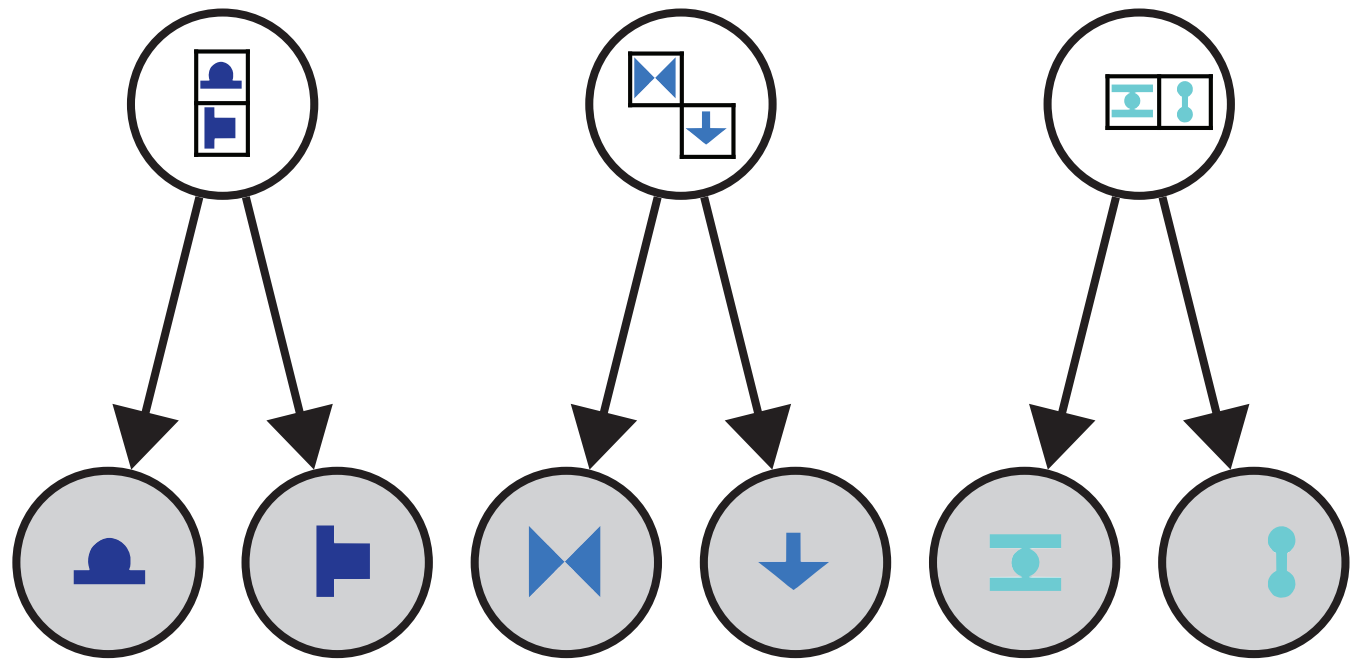


# ALTERNATIVE THEORIES

associative learning



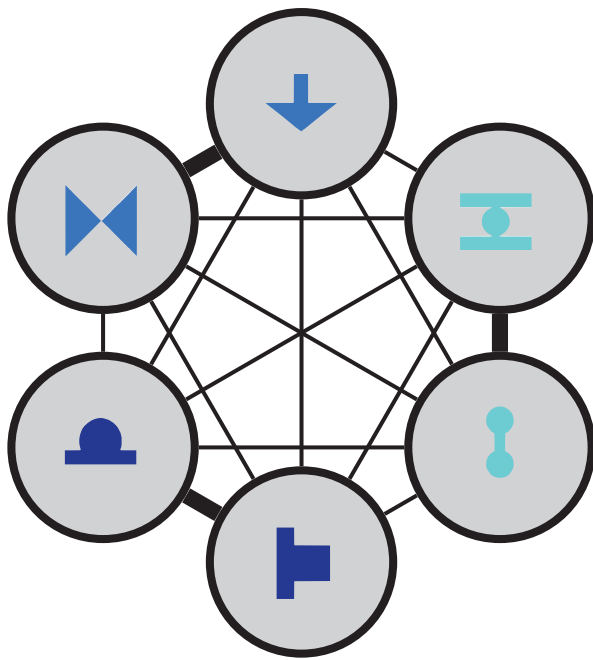
Bayesian learning



*Orbán & al, PNAS 2008*

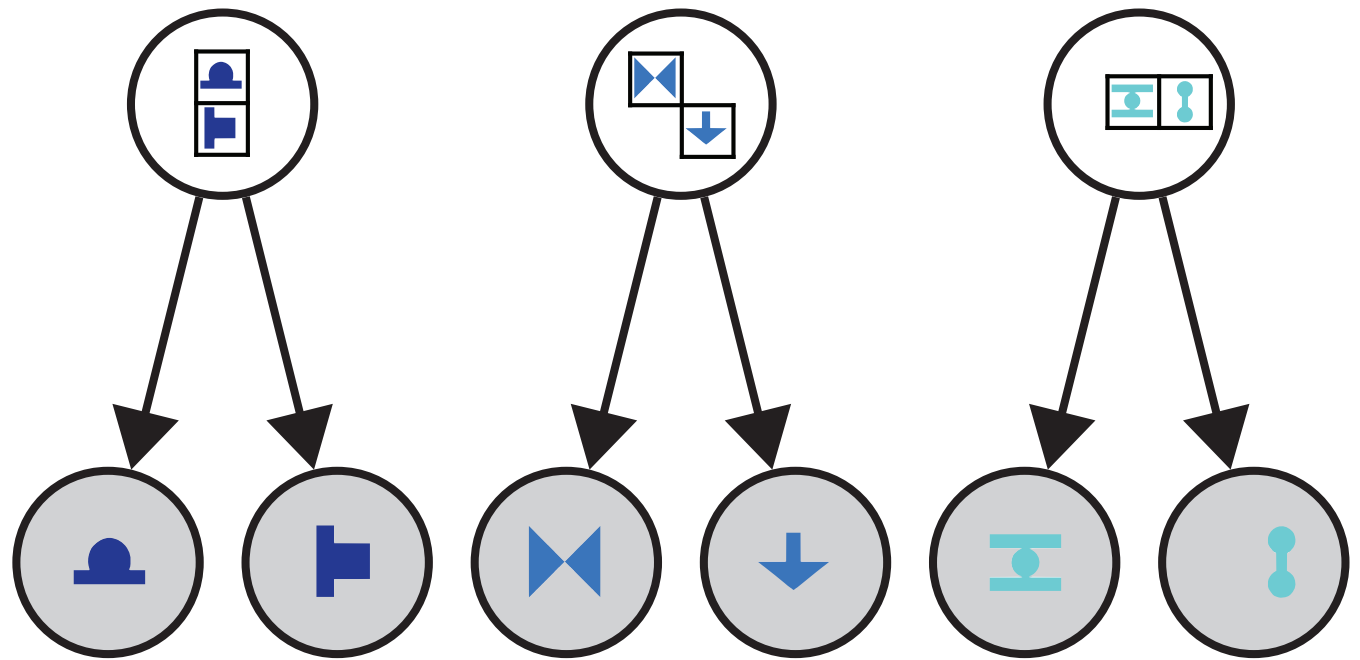
# ALTERNATIVE THEORIES

associative learning



Boltzmann machine  
+ Gaussian Markov random field

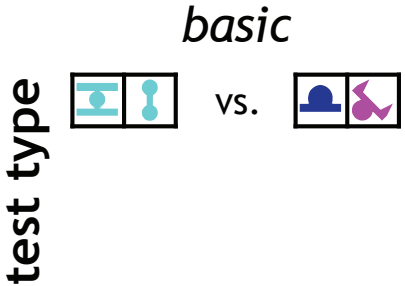
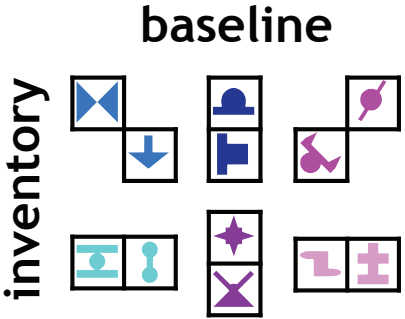
Bayesian learning



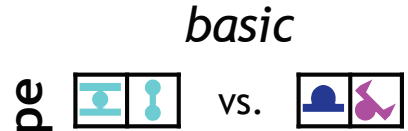
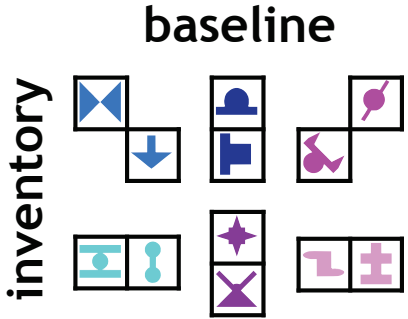
sigmoid belief network  
+ product of (conditional) Gaussian experts

*Orbán & al, PNAS 2008*

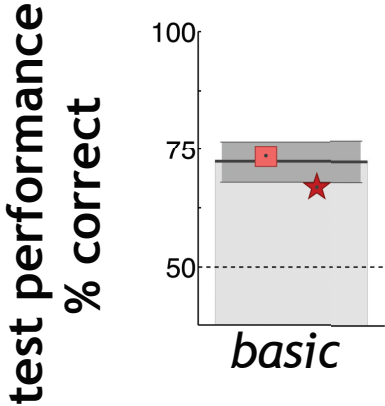
# MULTIPLE EXPERIMENTS



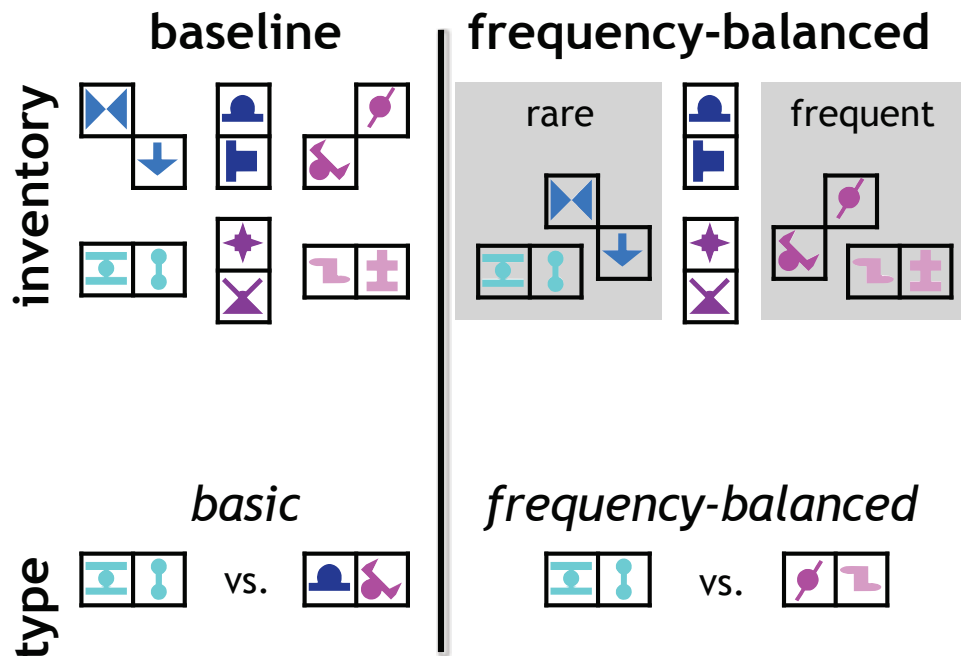
# MULTIPLE EXPERIMENTS



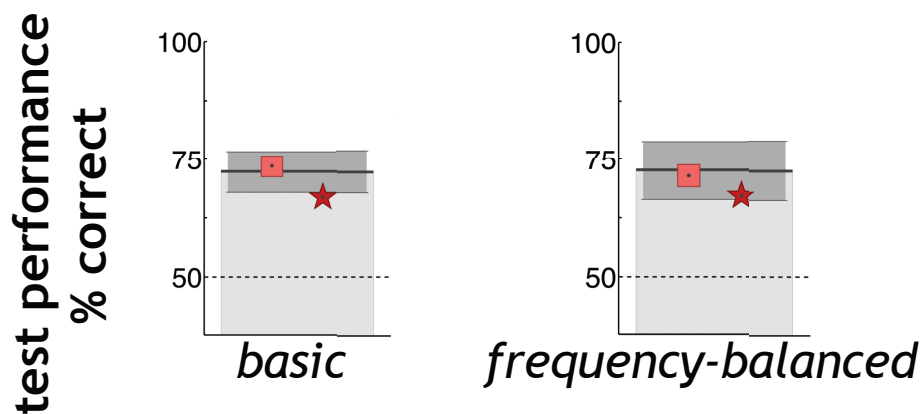
- humans
- associative learner
- Bayesian learner



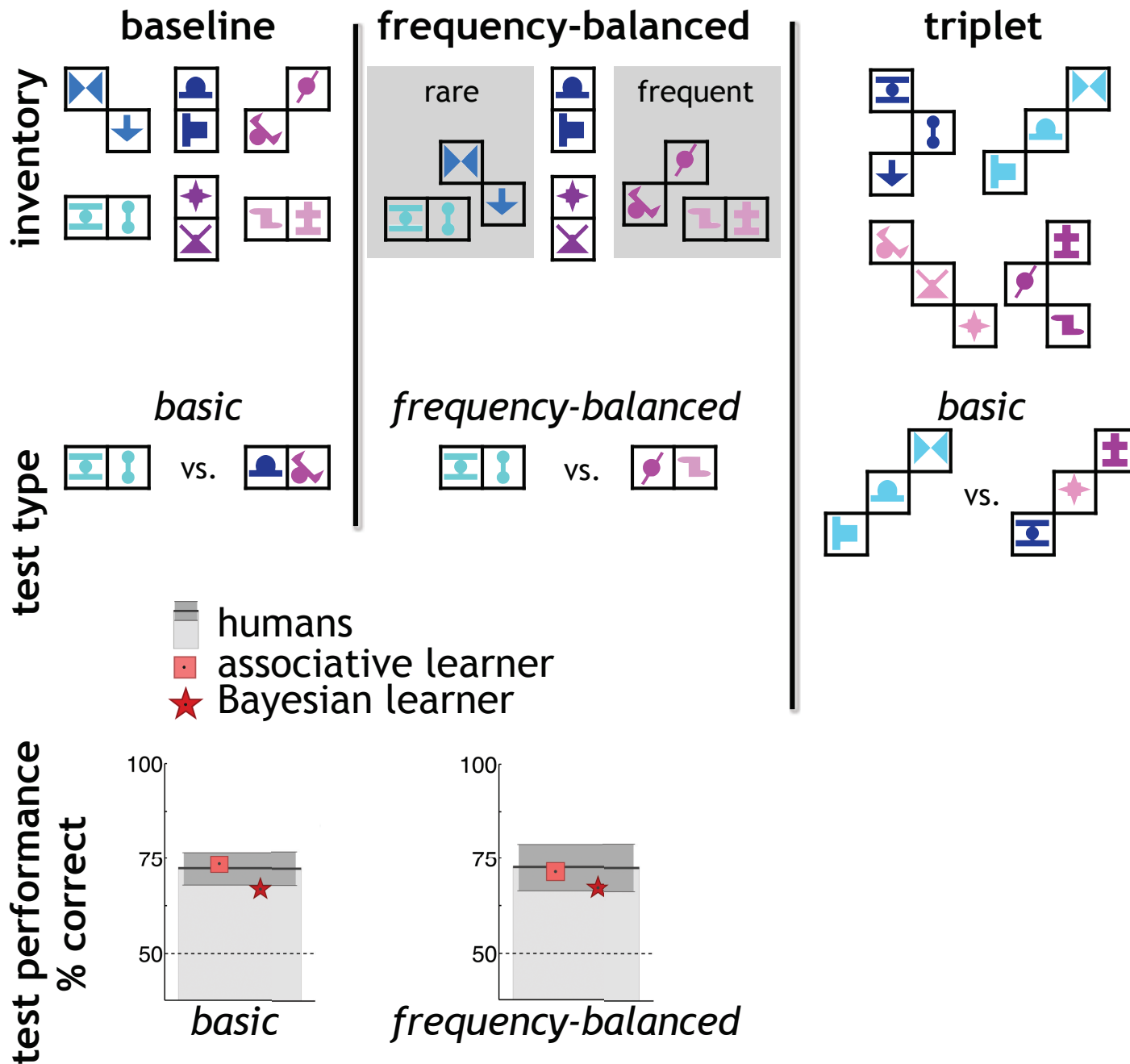
# MULTIPLE EXPERIMENTS



-  humans
-  associative learner
-  Bayesian learner

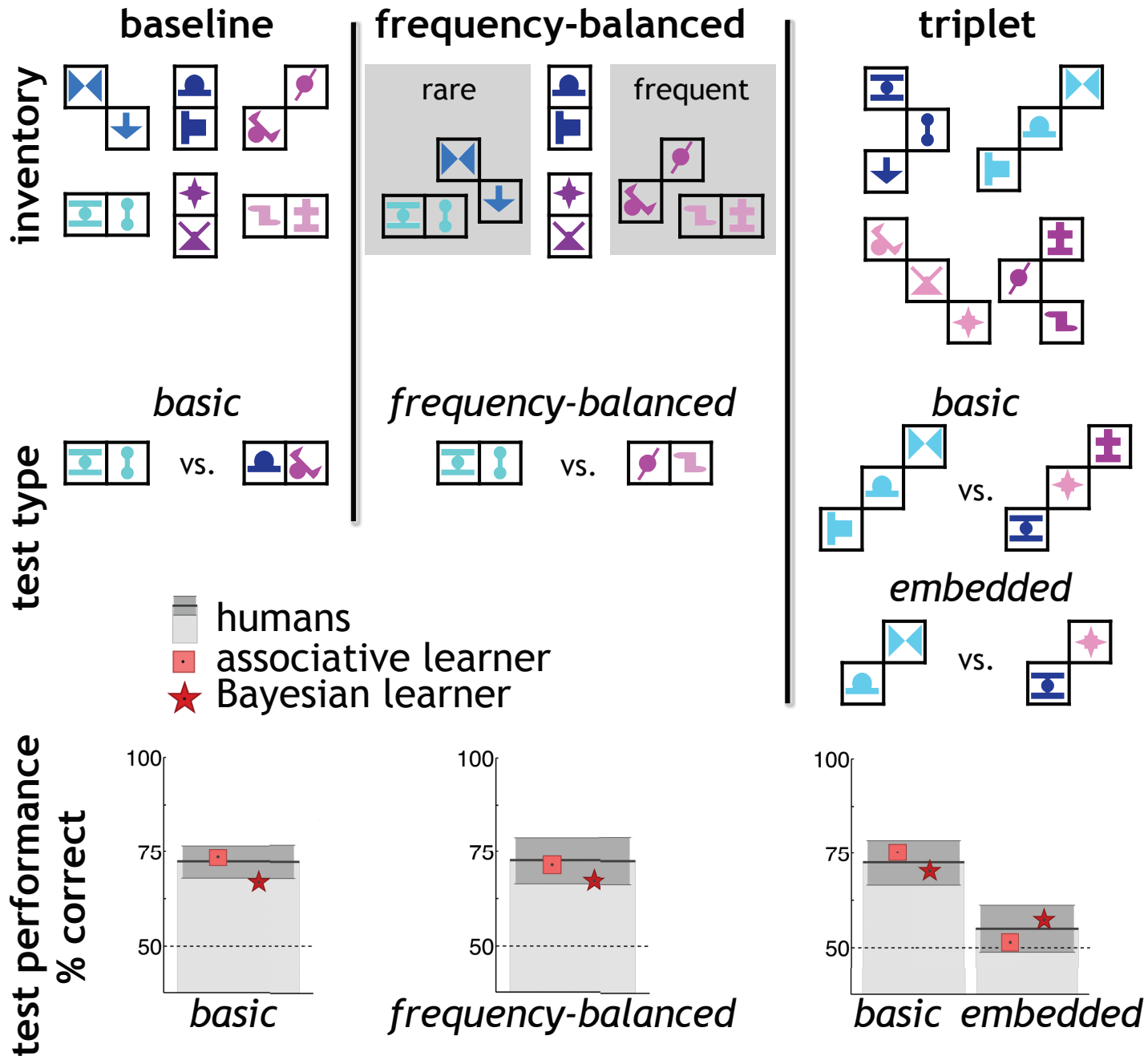


# MULTIPLE EXPERIMENTS

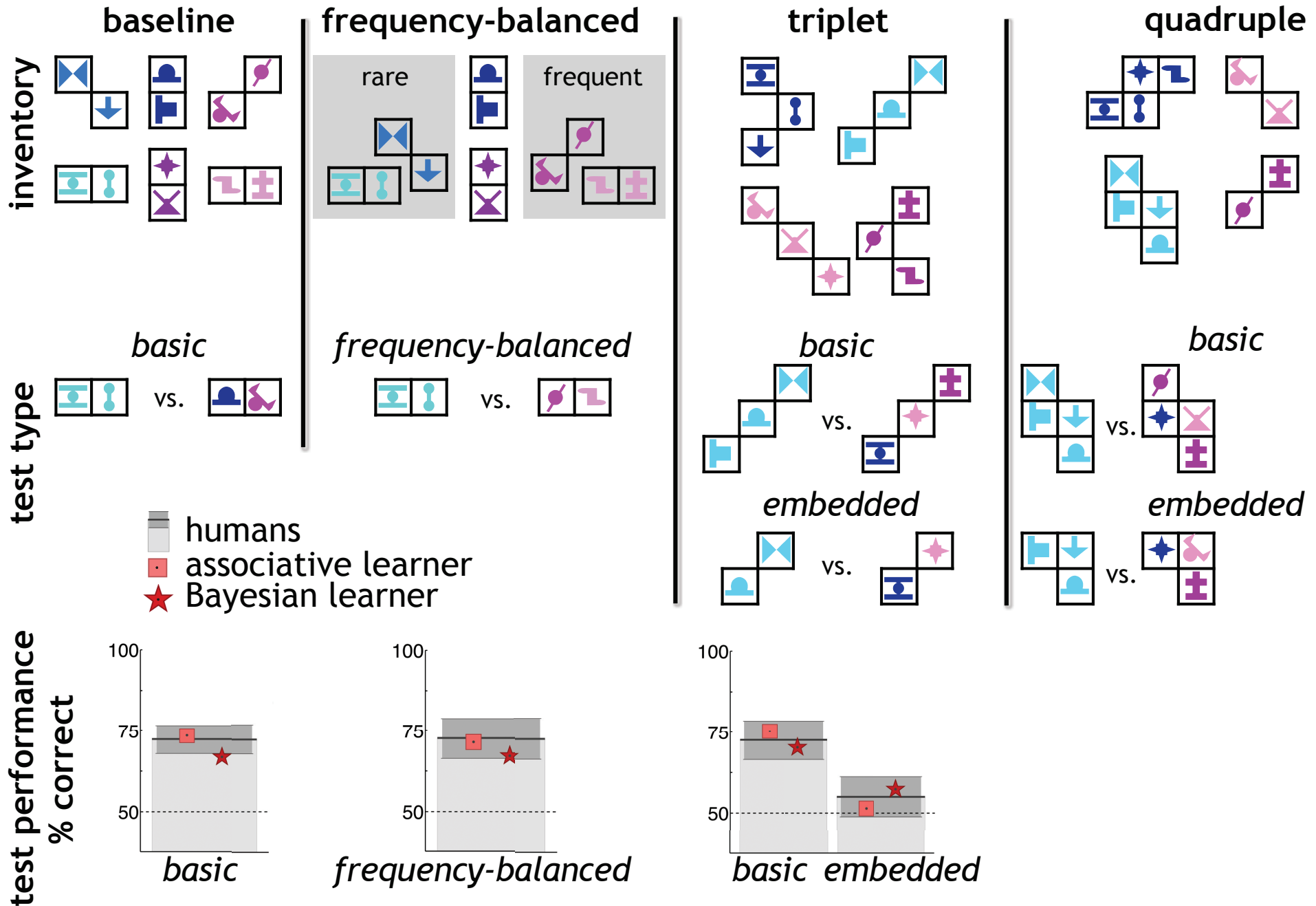




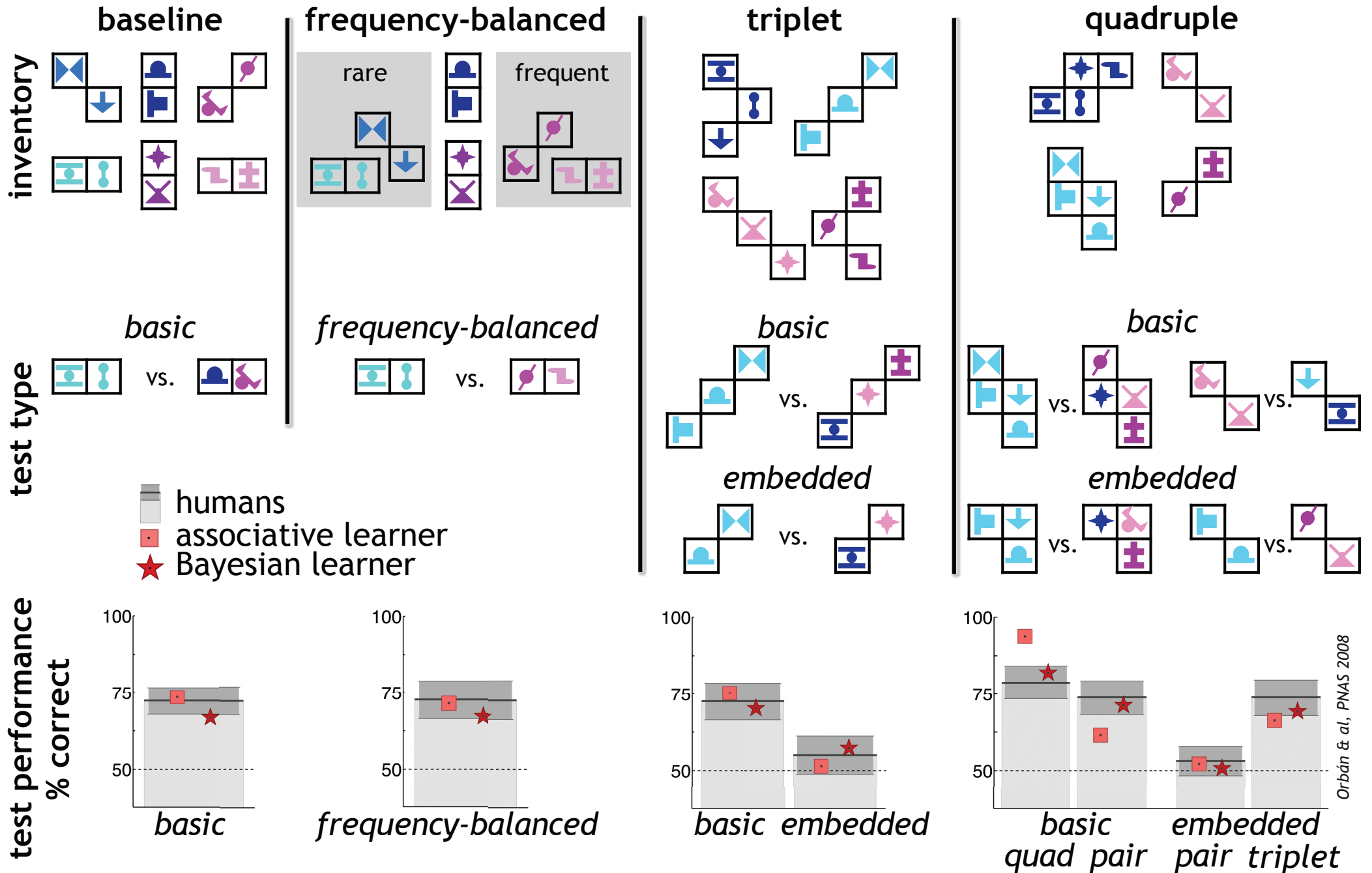
# MULTIPLE EXPERIMENTS



# MULTIPLE EXPERIMENTS

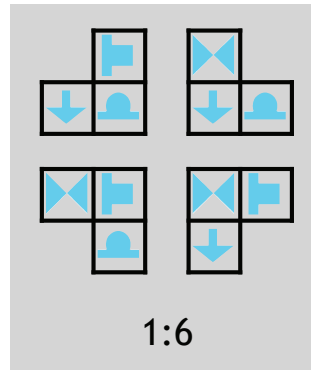


# MULTIPLE EXPERIMENTS



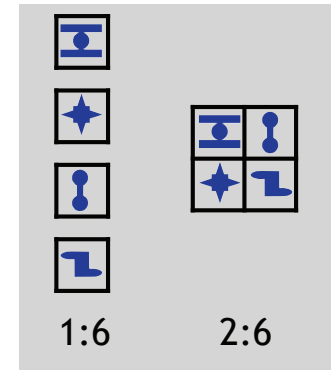
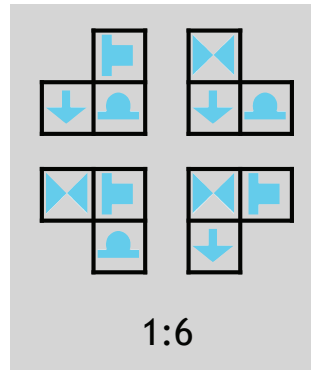
# ASSOCIATIVE VS. BAYESIAN LEARNING

inventory:



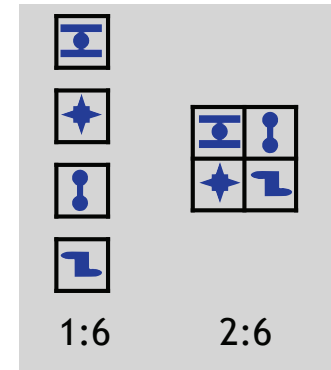
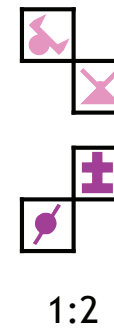
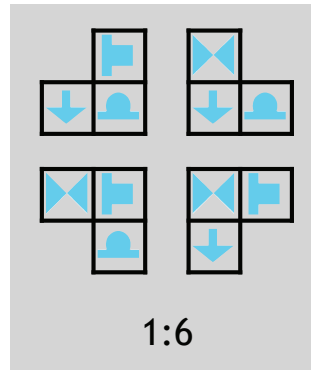
# ASSOCIATIVE VS. BAYESIAN LEARNING

inventory:



# ASSOCIATIVE VS. BAYESIAN LEARNING

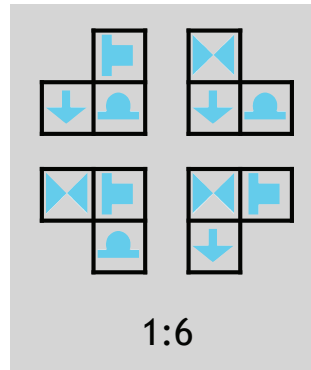
inventory:



# ASSOCIATIVE VS. BAYESIAN LEARNING

inventory:

1<sup>st</sup> order statistic:  
shape frequencies

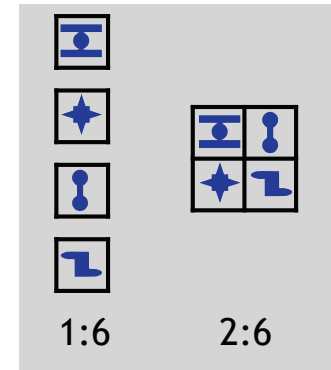


1:6

$$3 \times 1/6$$



1:2



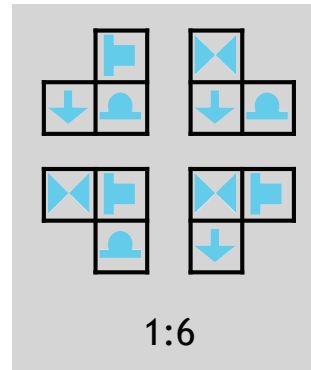
1:6

2:6

$$1/6 + 2/6$$

# ASSOCIATIVE VS. BAYESIAN LEARNING

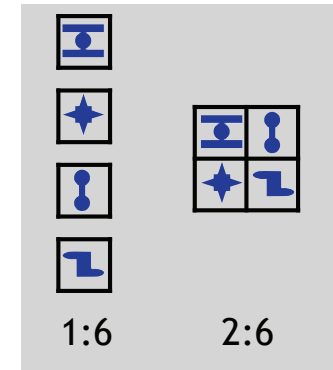
inventory:



1:6



1:2



1:6

2:6

1<sup>st</sup> order statistic:  
shape frequencies

$$3 \times 1/6$$

$$1/6 + 2/6$$

2<sup>nd</sup> order statistic:  
pairwise correlations

both present

$$2 \times 1/6$$

$$2/6$$

both absent

$$2/6$$

$$2 \times 1/6$$

one present

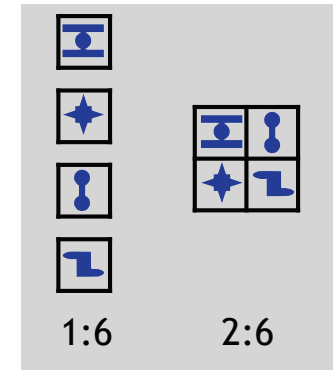
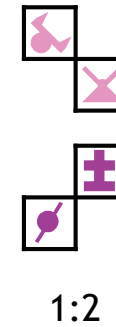
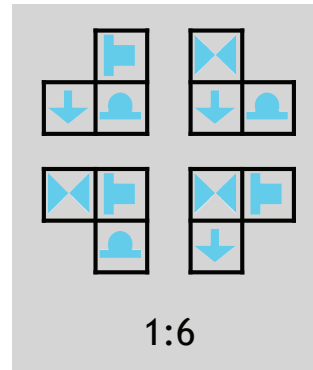
$$2 \times 1/6$$

$$2 \times 1/6$$



# ASSOCIATIVE VS. BAYESIAN LEARNING

inventory:



1<sup>st</sup> order statistic:  
shape frequencies

$$3 \times 1/6$$

$$1/6 + 2/6$$

2<sup>nd</sup> order statistic:  
pairwise correlations

both present

$$2 \times 1/6$$

$$2/6$$

both absent

$$2/6$$

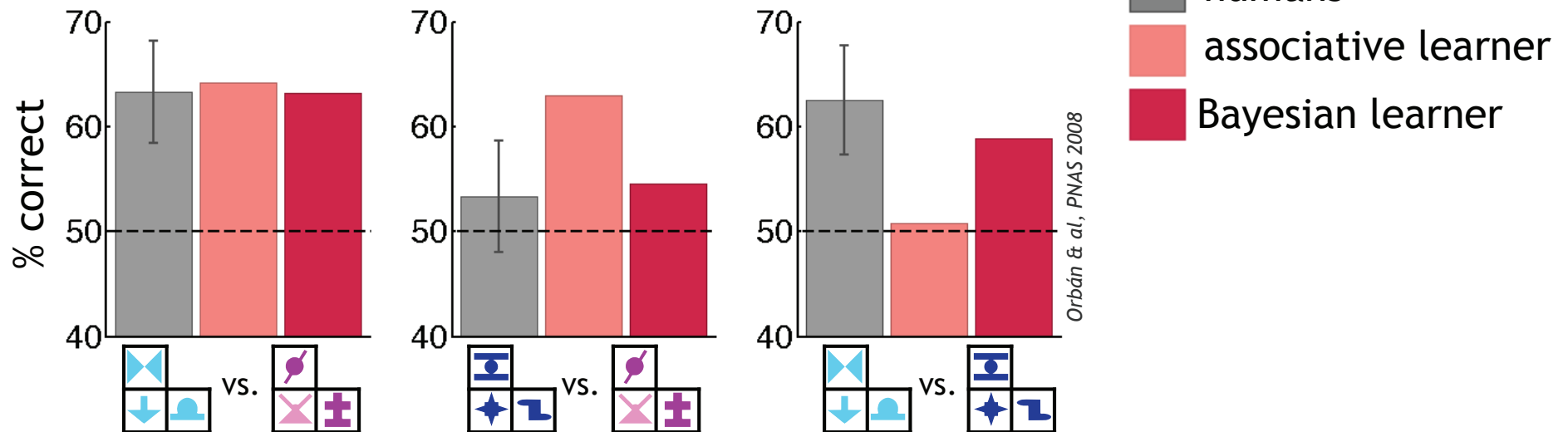
$$2 \times 1/6$$

one present

$$2 \times 1/6$$

$$2 \times 1/6$$

test performance:



# STATISTICAL CHUNKS = OBJECTS?



Gábor  
Lengyel



József  
Fiser



Alex  
Pantelides



Goda  
Zalalyte



James  
Ingram



Daniel  
Wolpert

# STATISTICAL CHUNKS = OBJECTS?

inventory

*pairs*



# STATISTICAL CHUNKS = OBJECTS?

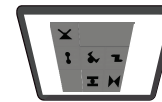
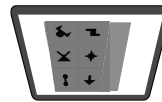
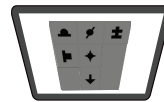
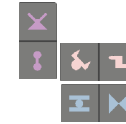
inventory

exposure

*pairs*



*objects defined by visual stats*

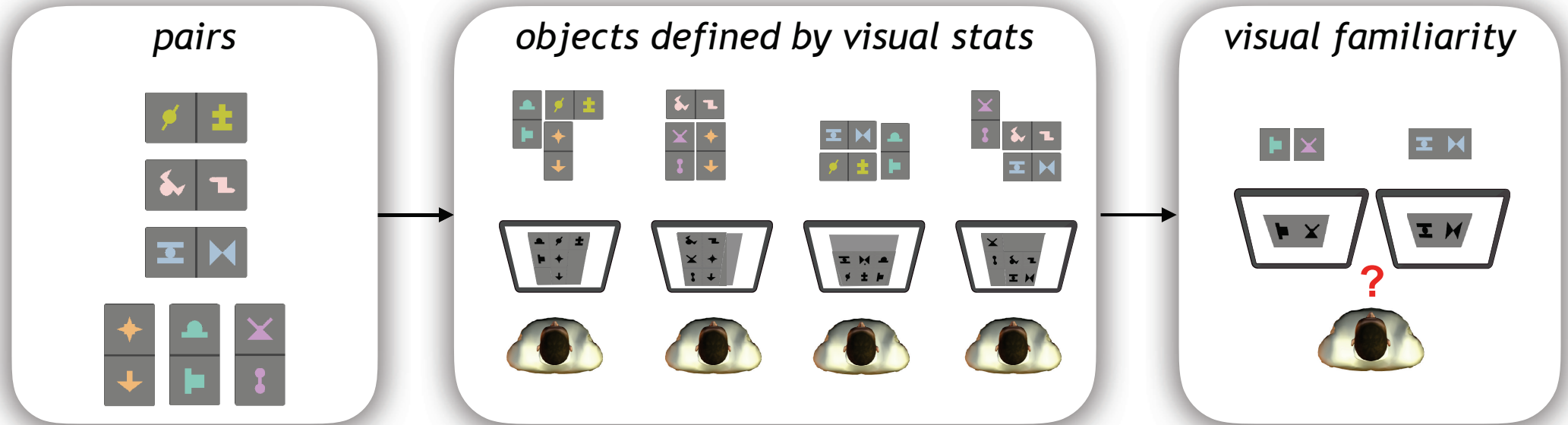


# STATISTICAL CHUNKS = OBJECTS?

inventory

exposure

test



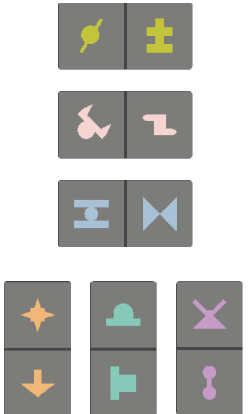
# STATISTICAL CHUNKS = OBJECTS?

inventory

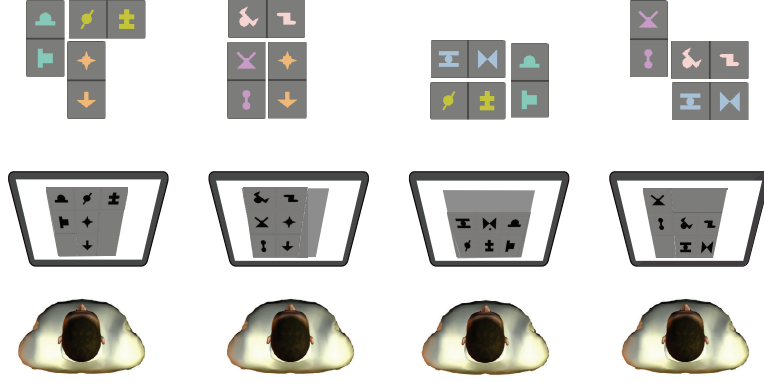
exposure

test

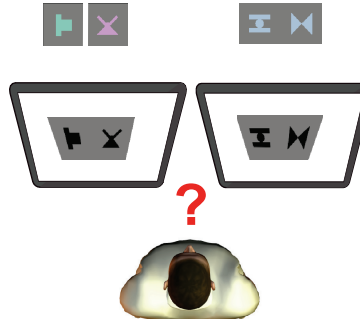
*pairs*



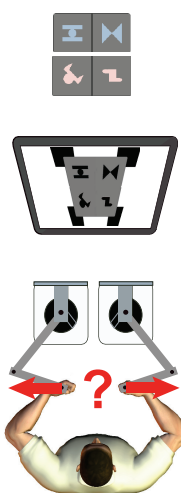
*objects defined by visual stats*



*visual familiarity*



*haptic pulling*



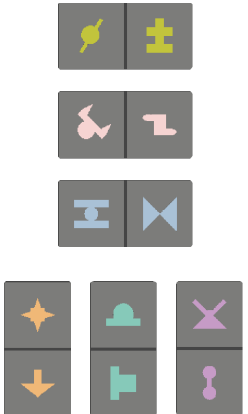
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inventory

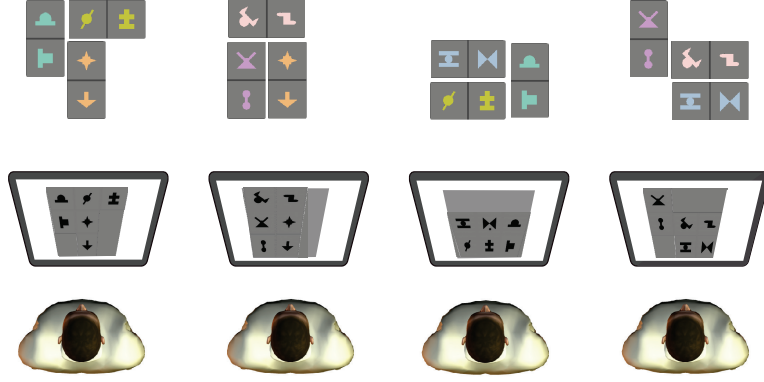
exposure

test

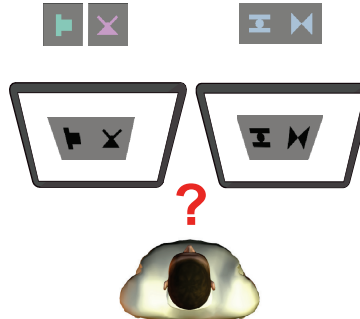
*pairs*



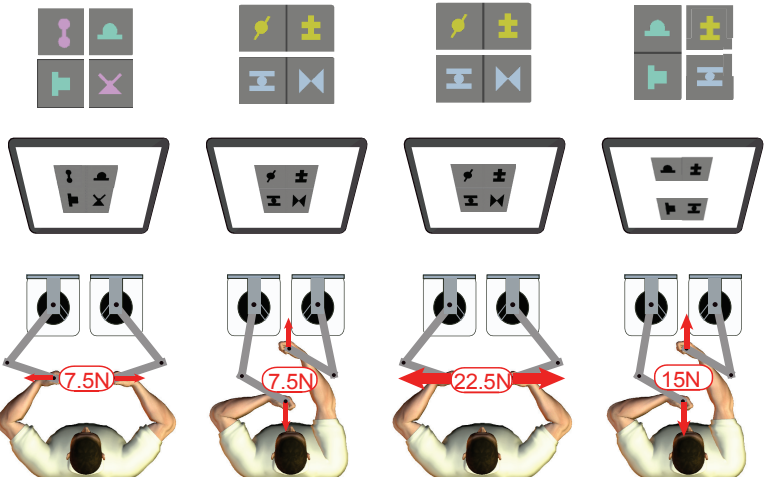
*objects defined by visual stats*



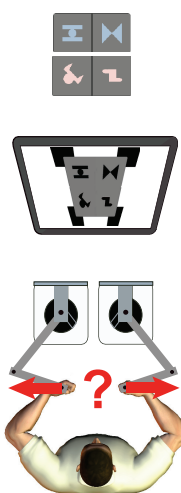
*visual familiarity*



*objects defined by haptic stats*



*haptic pulling*



Lengyel & al, under review

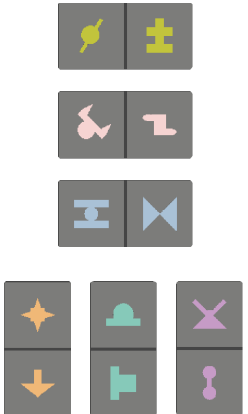
# STATISTICAL CHUNKS = OBJECTS?

inventory

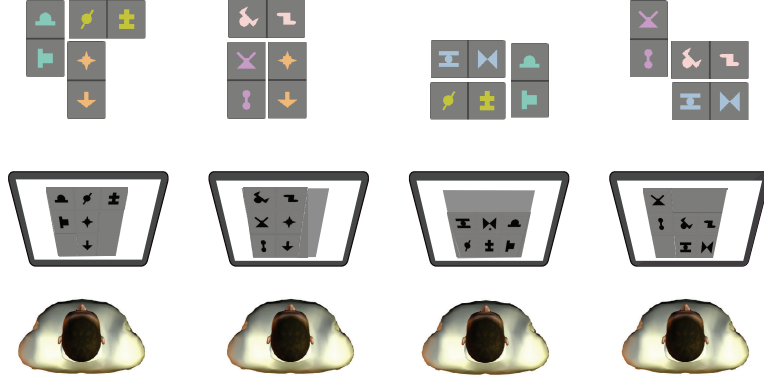
exposure

test

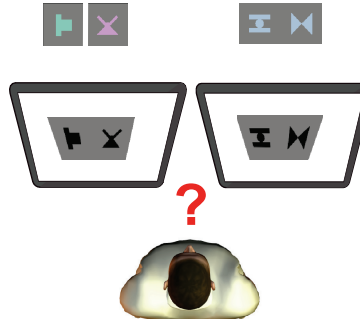
*pairs*



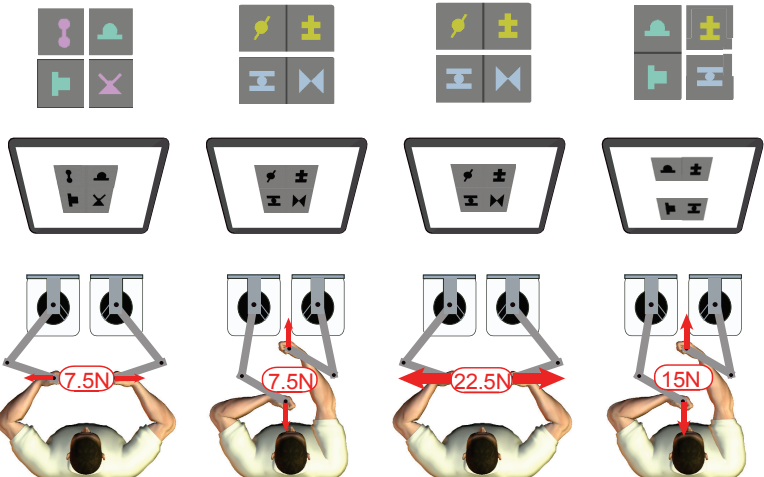
*objects defined by visual stats*



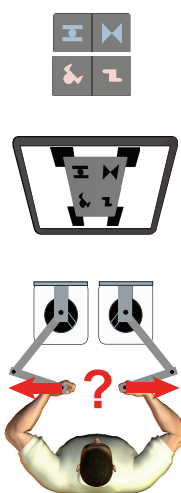
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*haptic pulling*



Lengyel & al, under review



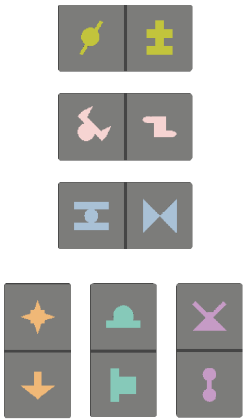
# STATISTICAL CHUNKS = OBJECTS?

inventory

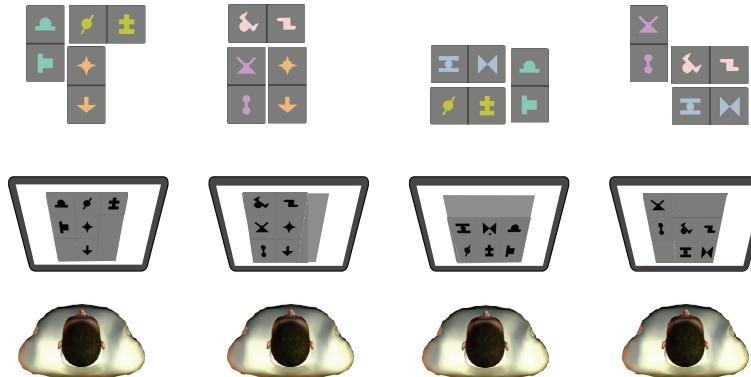
exposure

test

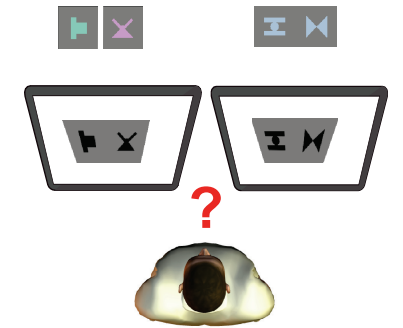
*pairs*



*objects defined by visual stats*

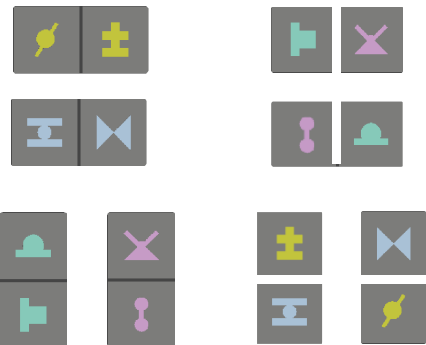


*visual familiarity*

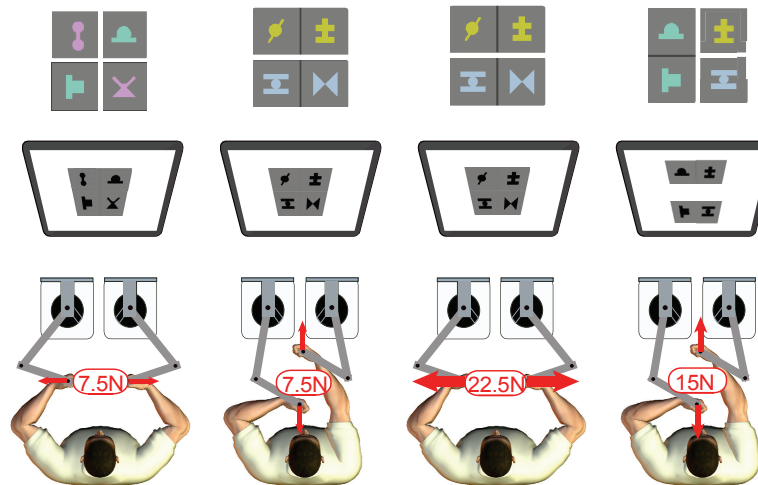


*true pairs*

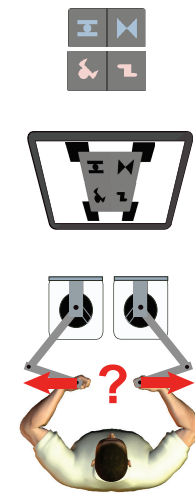
*pseudo pairs*



*objects defined by haptic stats*

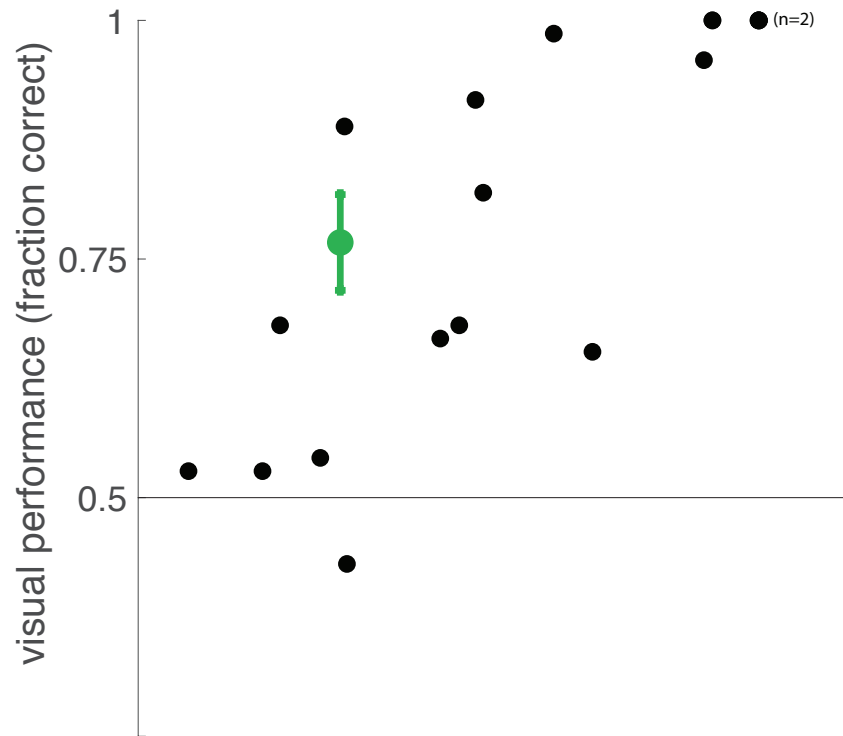


*haptic pulling*



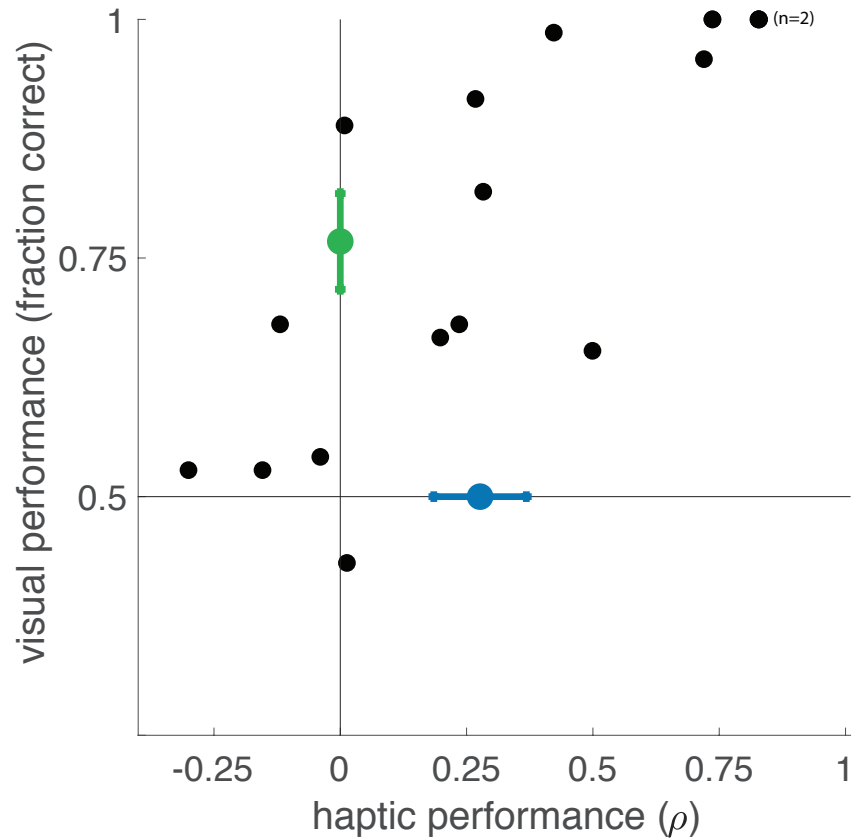
# ACROSS-MODALITY GENERALISATION

*objects defined by visual stats*



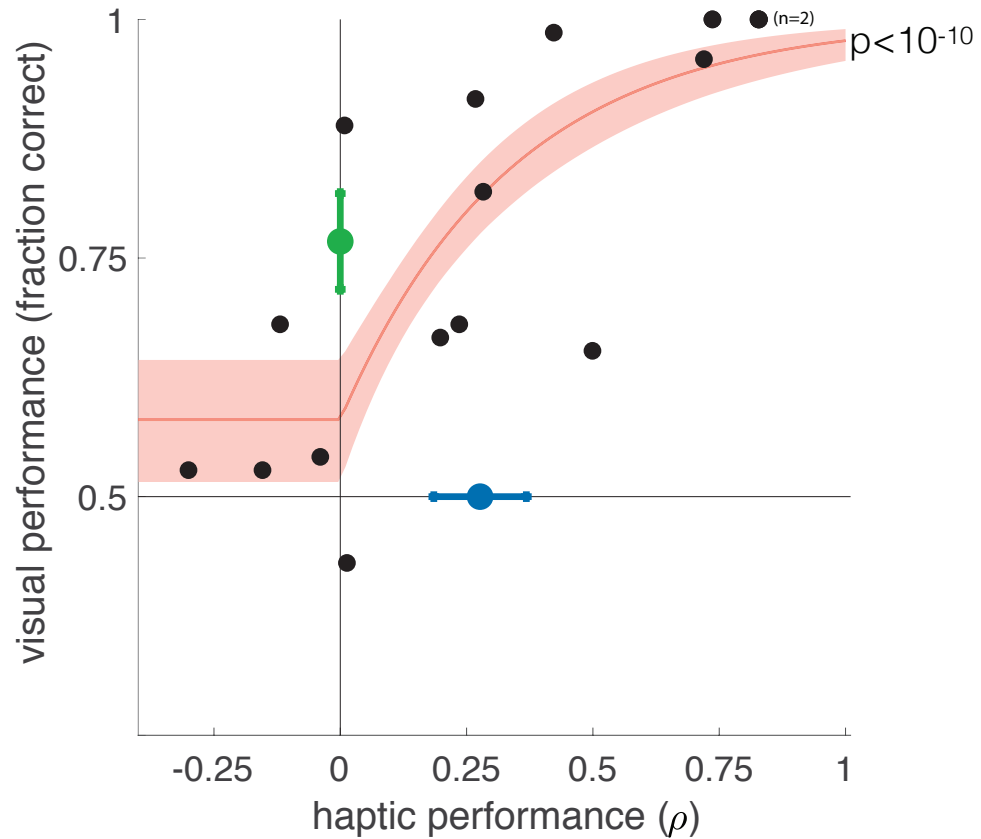
# ACROSS-MODALITY GENERALISATION

*objects defined by visual stats*



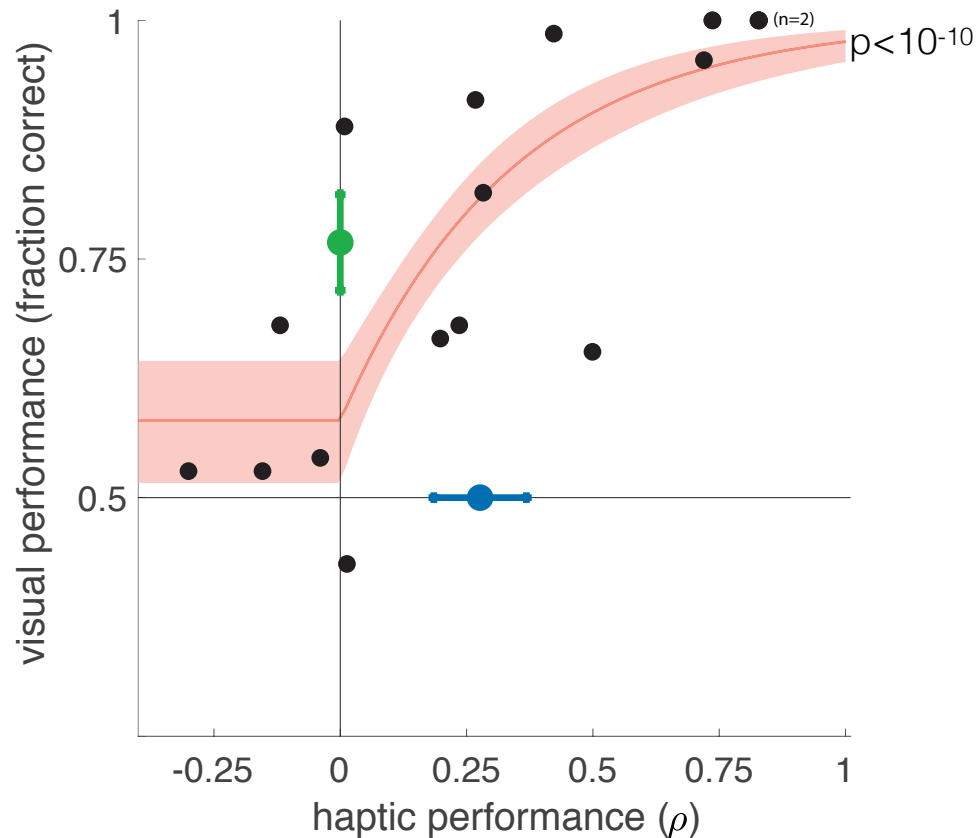
# ACROSS-MODALITY GENERALISATION

*objects defined by visual stats*

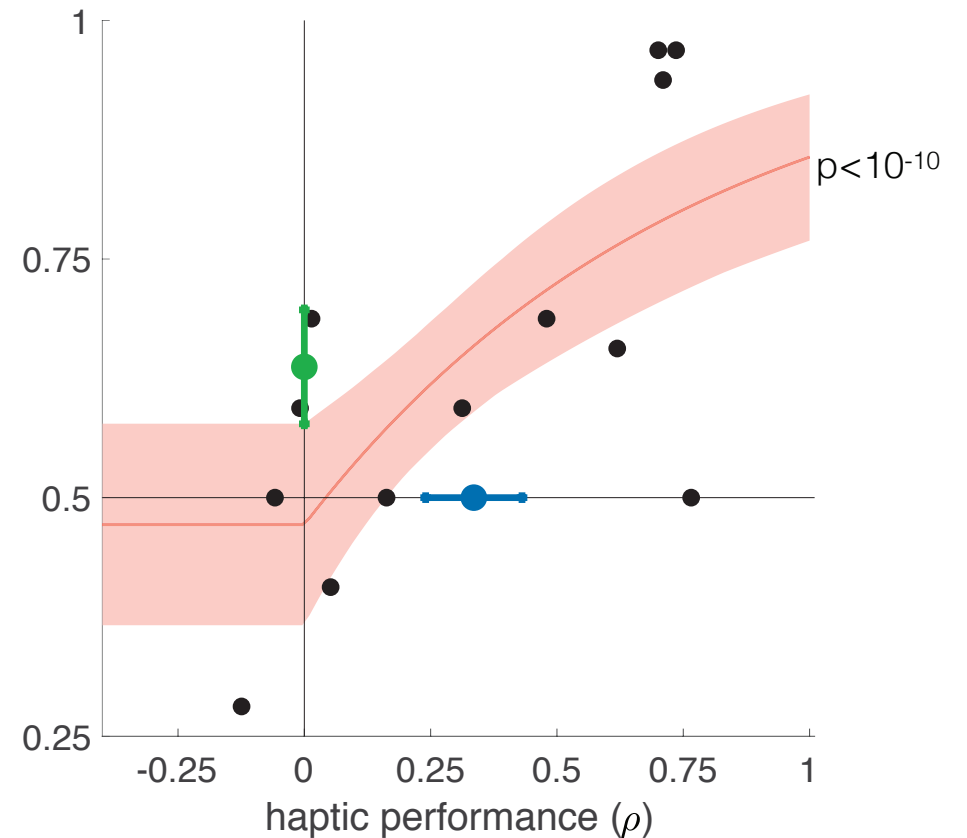


# ACROSS-MODALITY GENERALISATION

*objects defined by visual stats*



*objects defined by haptic stats*



*Lengyel & al, under review*

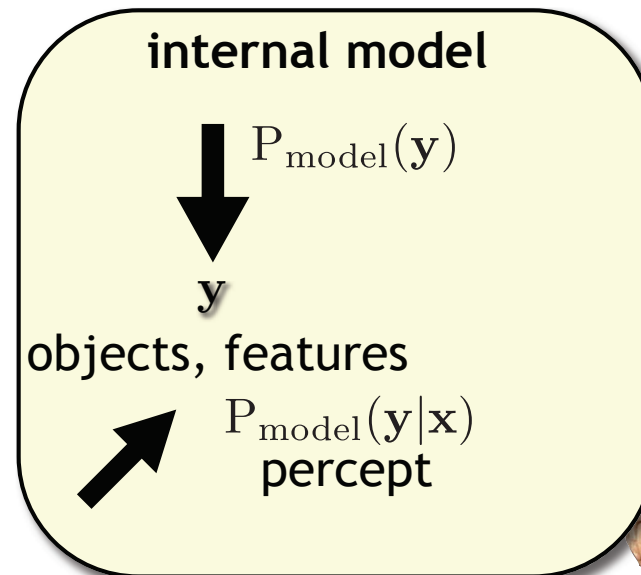
# PROBABILISTIC INTERNAL MODELS



$P_{\text{true}}(\mathbf{y})$   
↓  
 $\mathbf{y}$   
objects, features

$P_{\text{true}}(\mathbf{x}|\mathbf{y})$   
↘

$\mathbf{x}$   
sensory stimuli



## part 1. ADAPTATION TO ENVIRONMENT

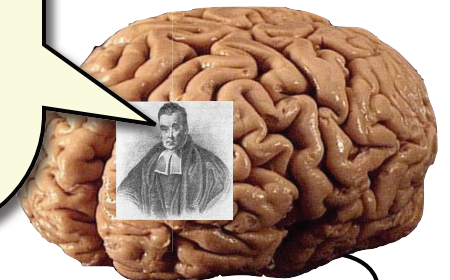
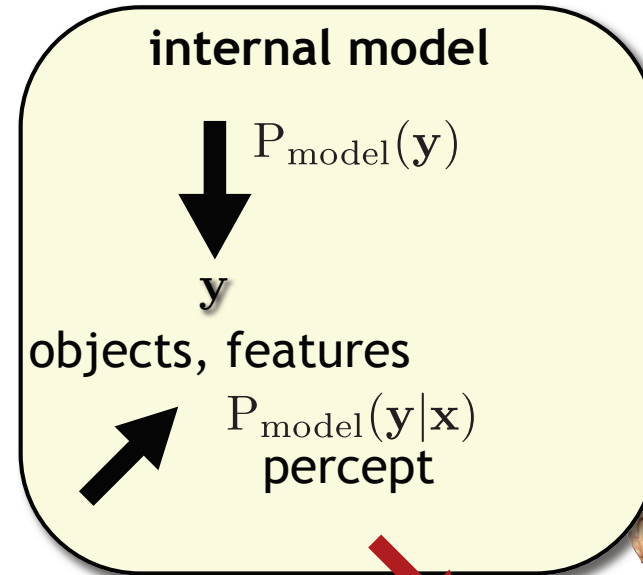
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$P_{\text{true}}(\mathbf{y})$   
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 $\mathbf{y}$   
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$P_{\text{true}}(\mathbf{x}|\mathbf{y})$   
↘

$\mathbf{x}$   
sensory stimuli



task → behaviour



part 1. ADAPTATION TO ENVIRONMENT

part 2. TASK-INDEPENDENCE

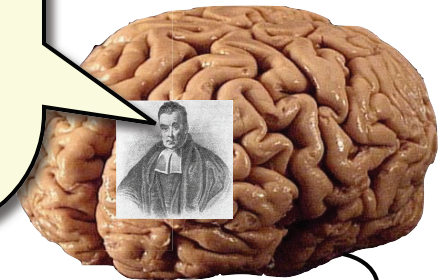
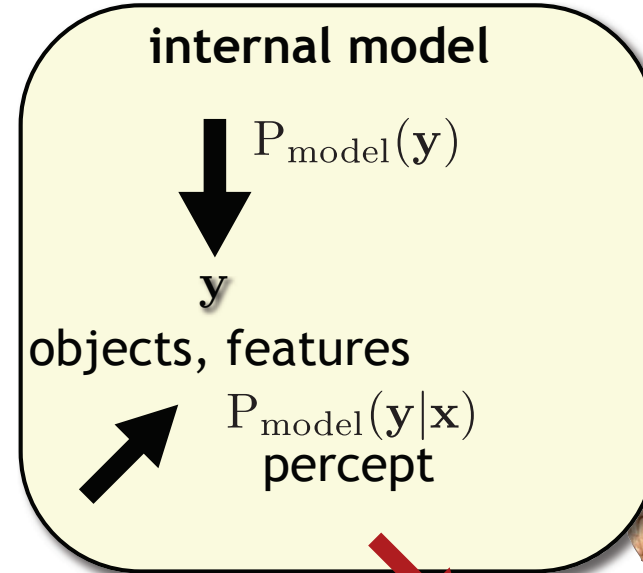
# PROBABILISTIC INTERNAL MODELS



$P_{\text{true}}(\mathbf{y})$   
↓  
 $\mathbf{y}$   
objects, features

$P_{\text{true}}(\mathbf{x}|\mathbf{y})$   
↘

$\mathbf{x}$   
sensory stimuli



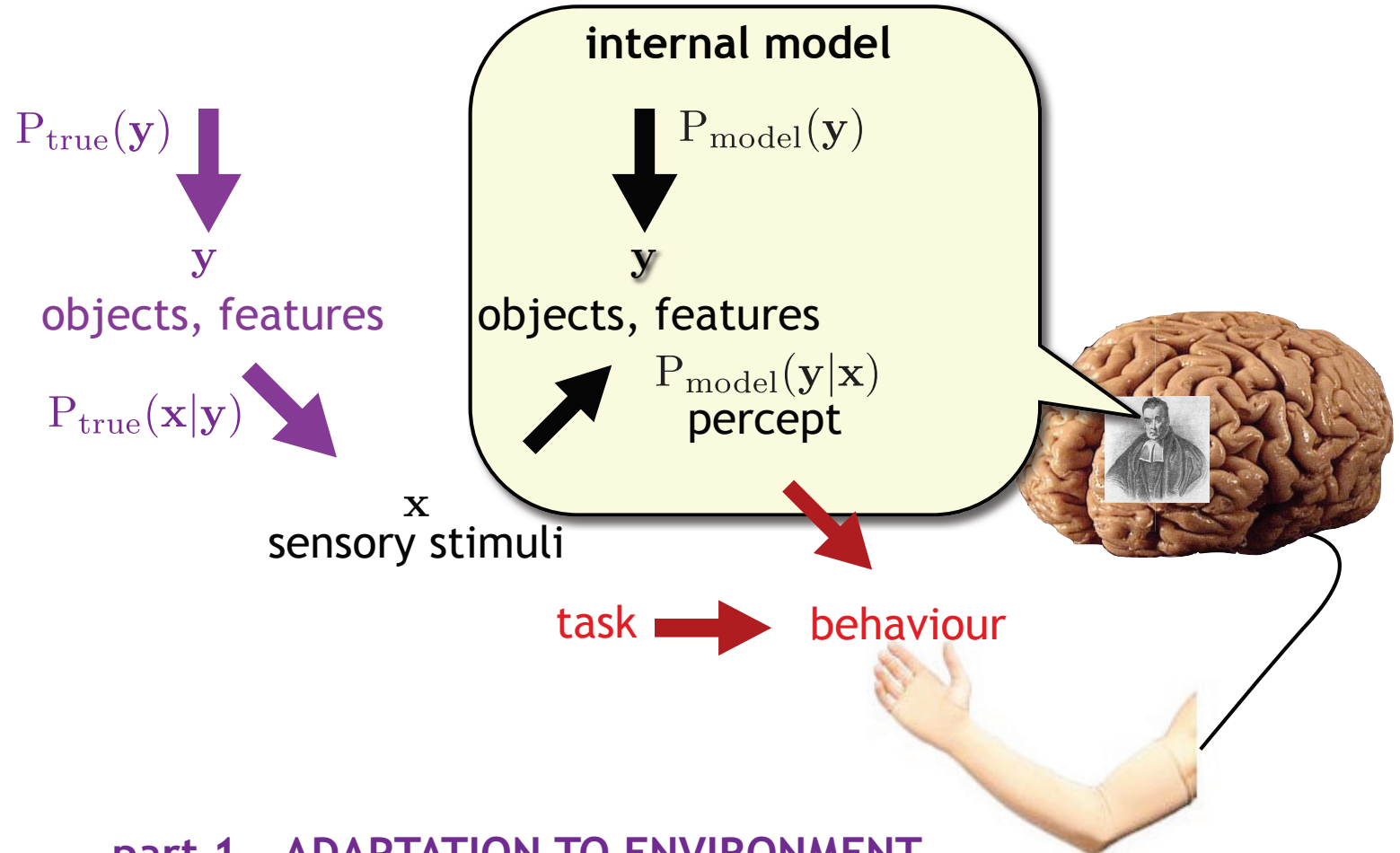
part 1. ADAPTATION TO ENVIRONMENT

part 2. TASK-INDEPENDENCE

issues with priors in single-task behavior:



# PROBABILISTIC INTERNAL MODELS



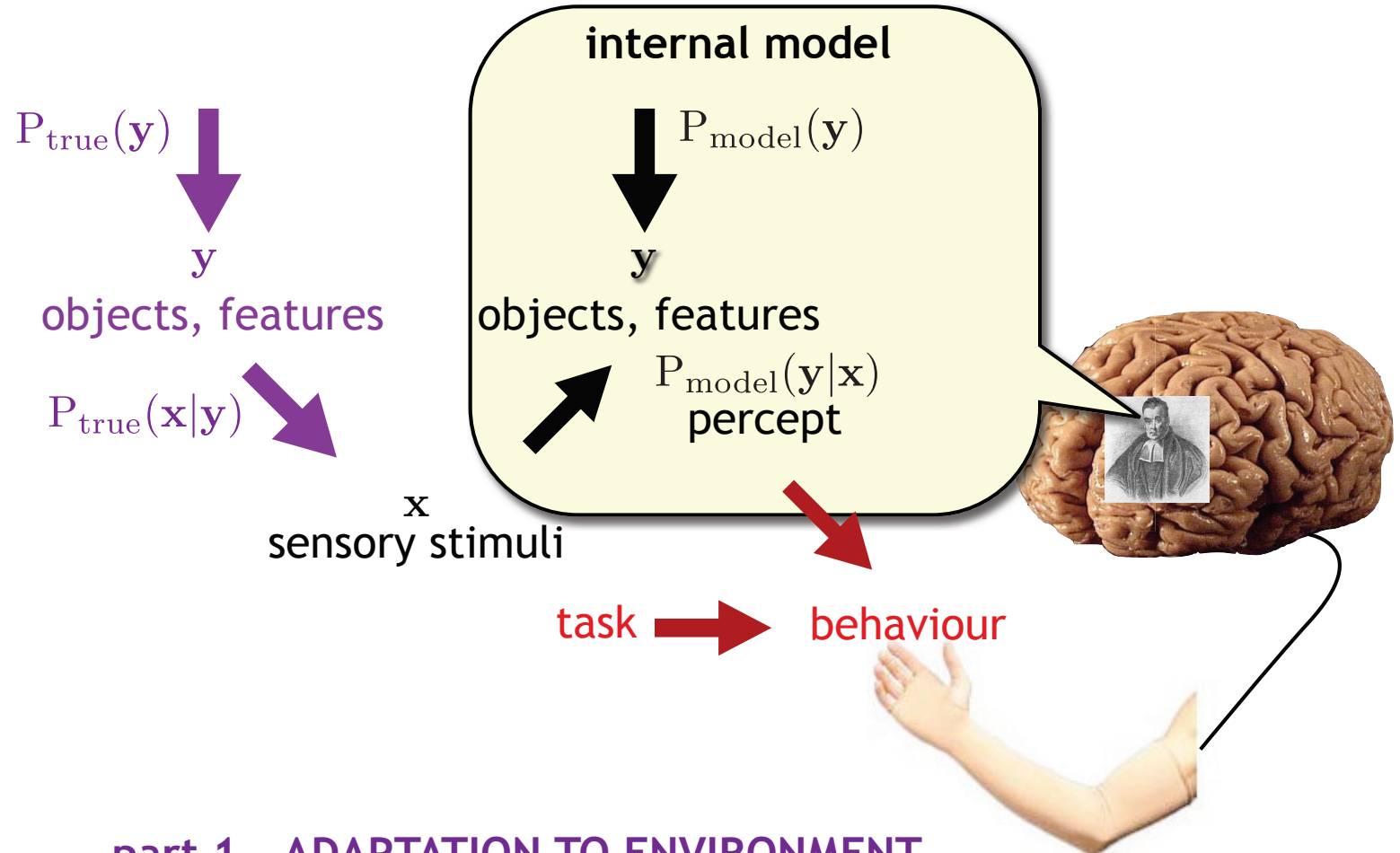
part 1. ADAPTATION TO ENVIRONMENT

part 2. TASK-INDEPENDENCE

issues with priors in single-task behavior:

► bunch of free parameters?

# PROBABILISTIC INTERNAL MODELS



part 1. ADAPTATION TO ENVIRONMENT

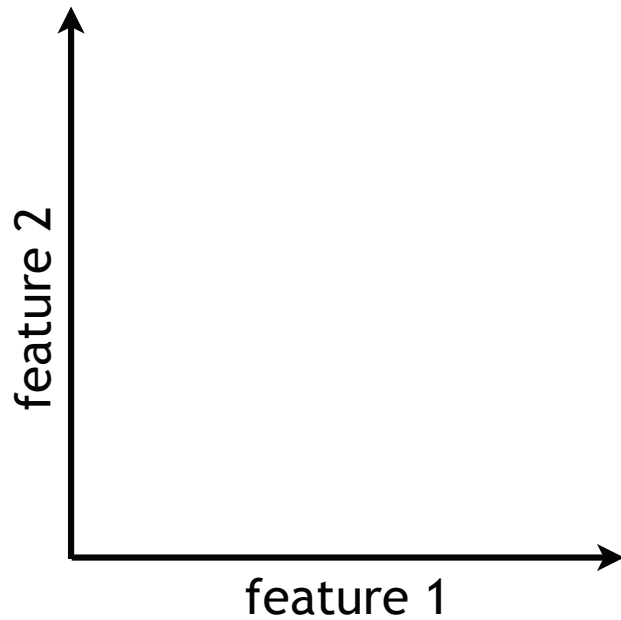
part 2. TASK-INDEPENDENCE

issues with priors in single-task behavior:

- ▶ bunch of free parameters?
- ▶ generalization across tasks?

# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE



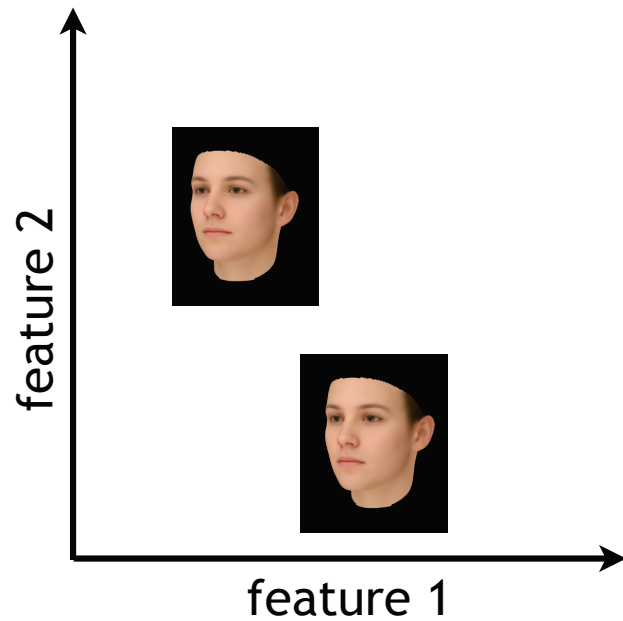
# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE



# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE



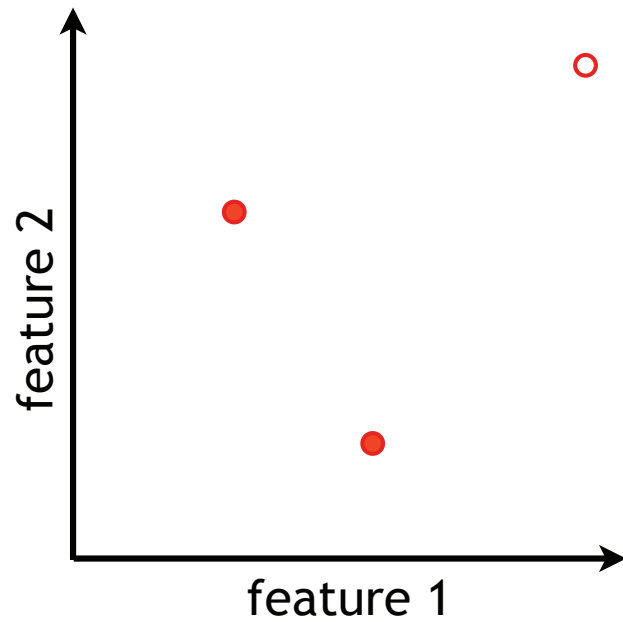
# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE



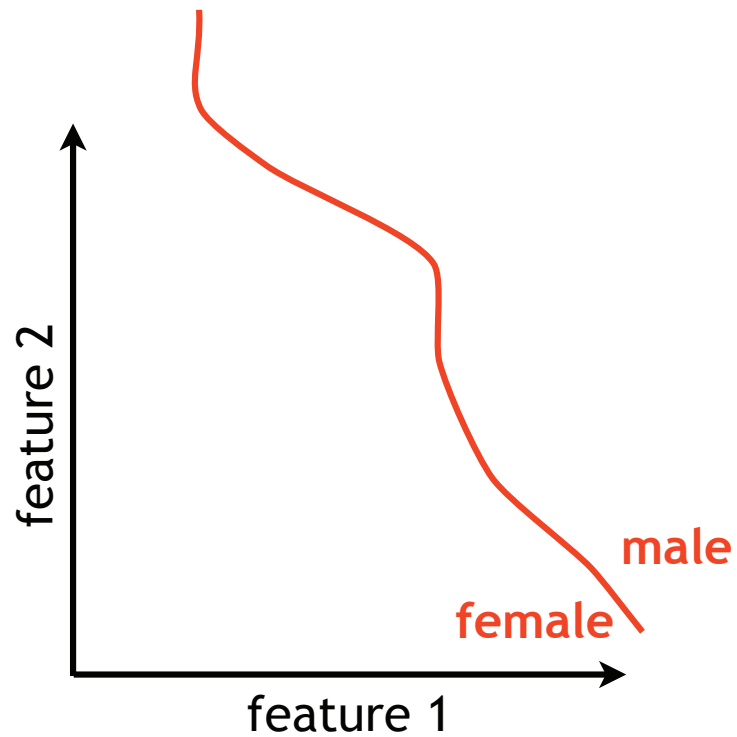
# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE



# DISCRIMINATIVE VS. GENERATIVE LEARNING

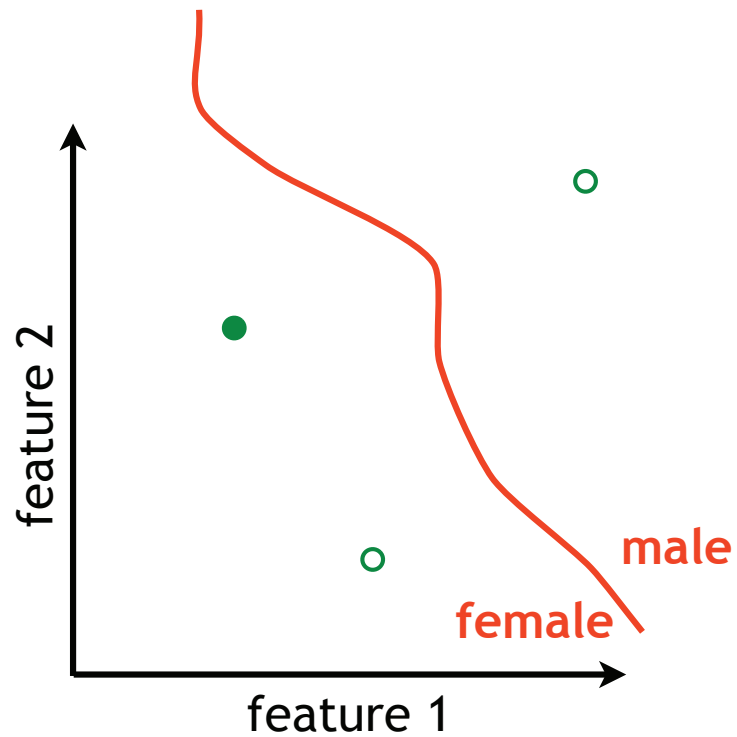
DISCRIMINATIVE





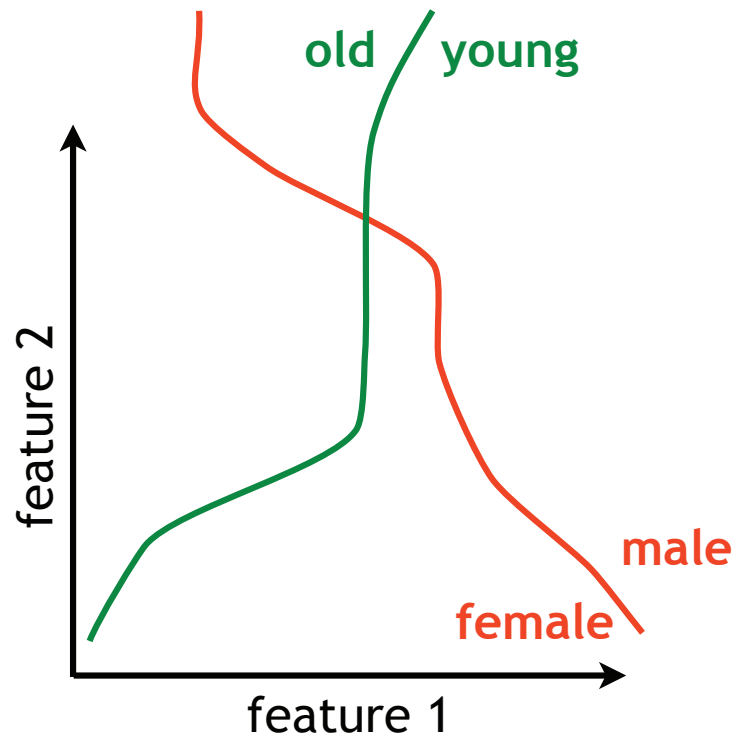
# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE



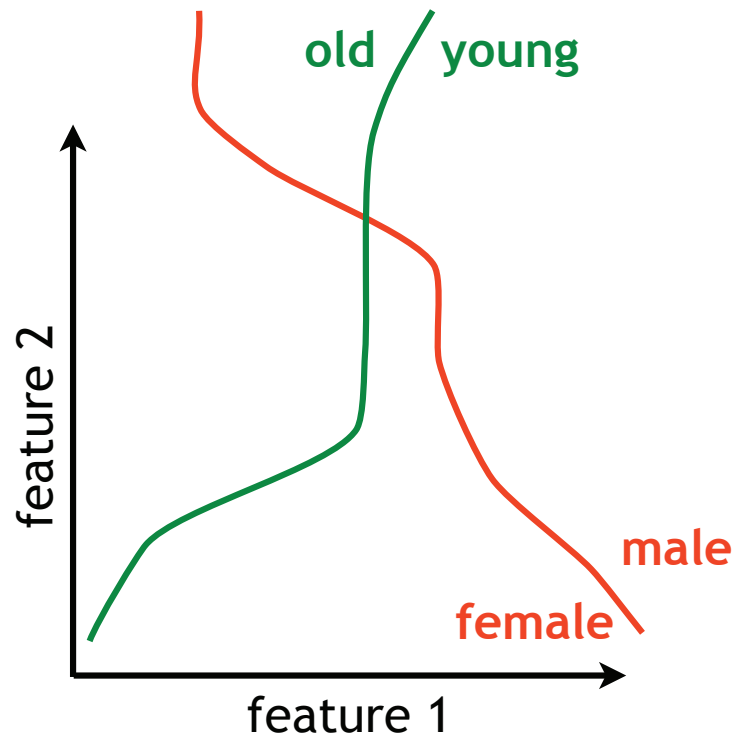
# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE

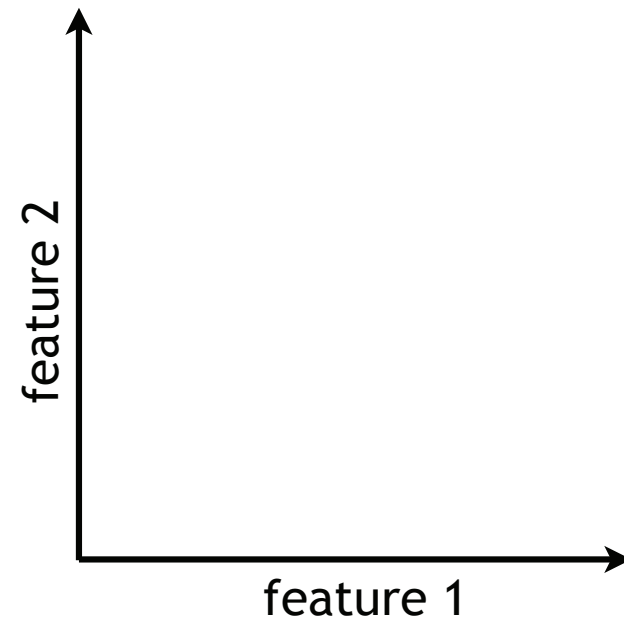


# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE

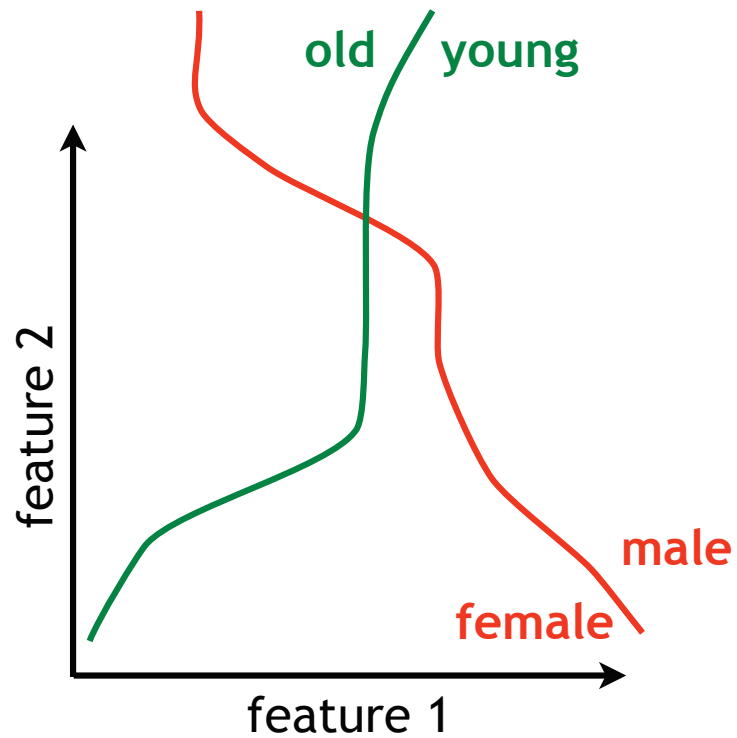


GENERATIVE

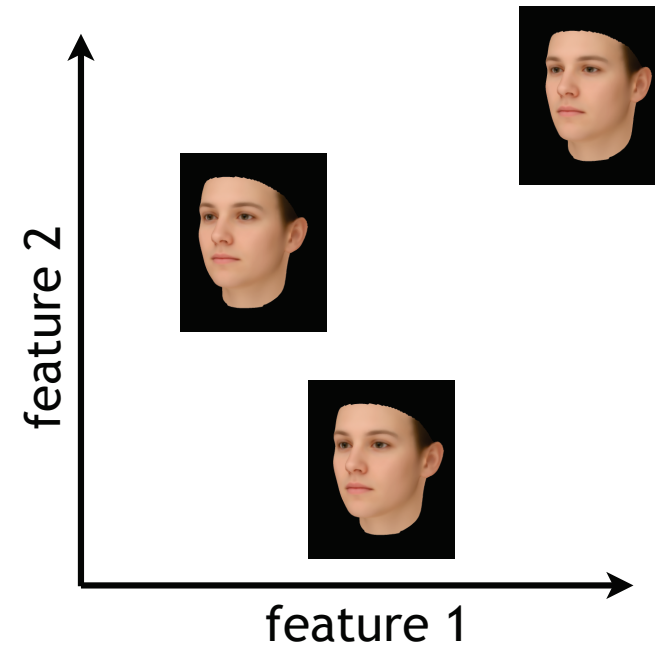


# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE

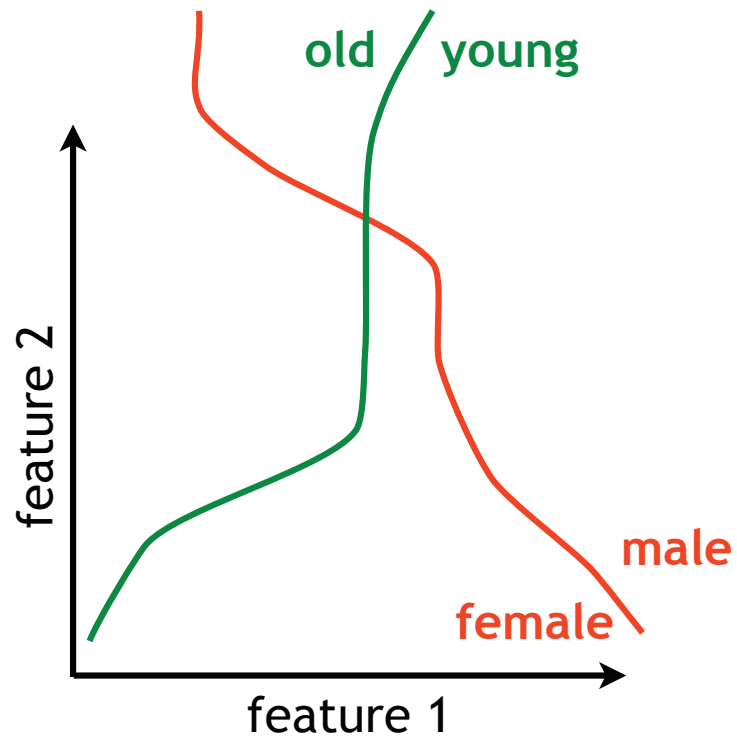


GENERATIVE

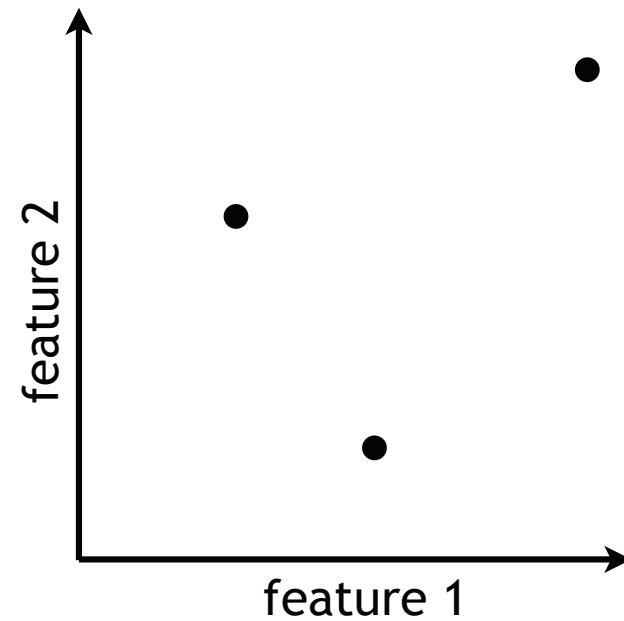


# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE

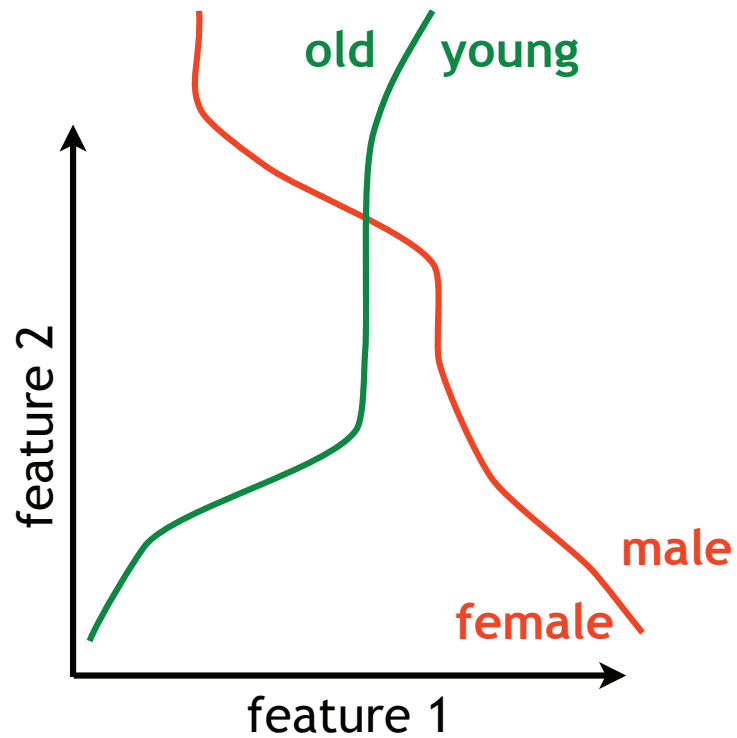


GENERATIVE

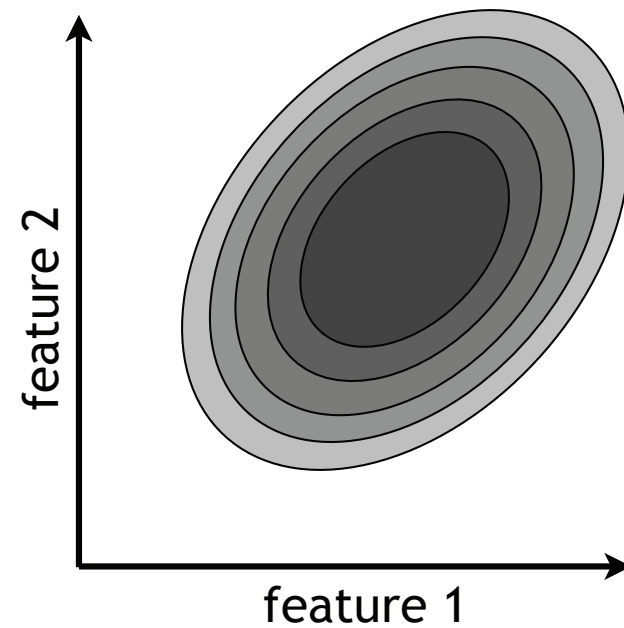


# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE

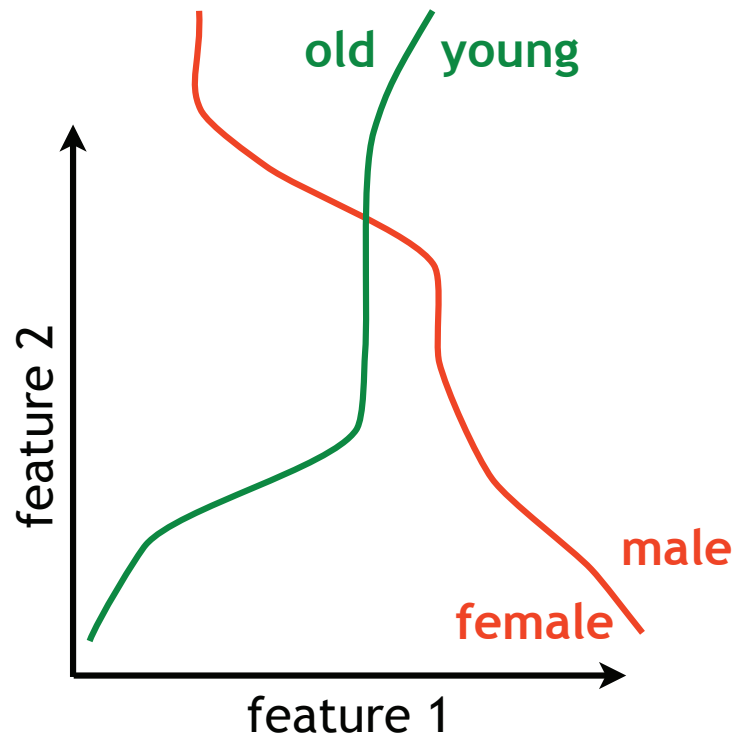


GENERATIVE

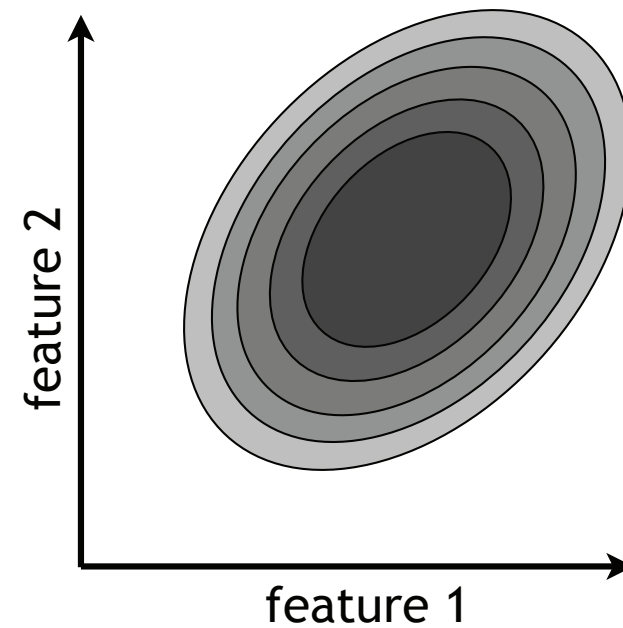


# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE  
*task-dependent representation*

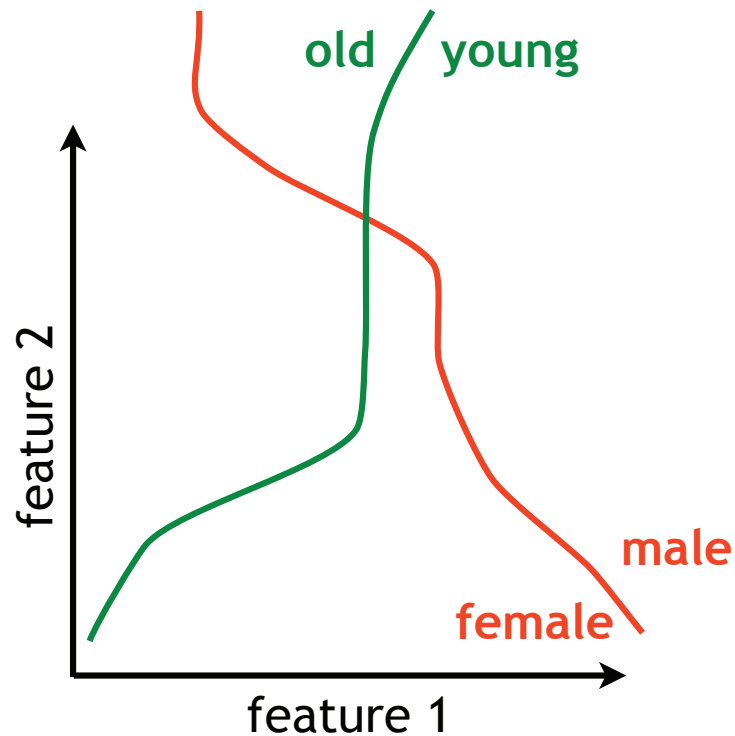


GENERATIVE

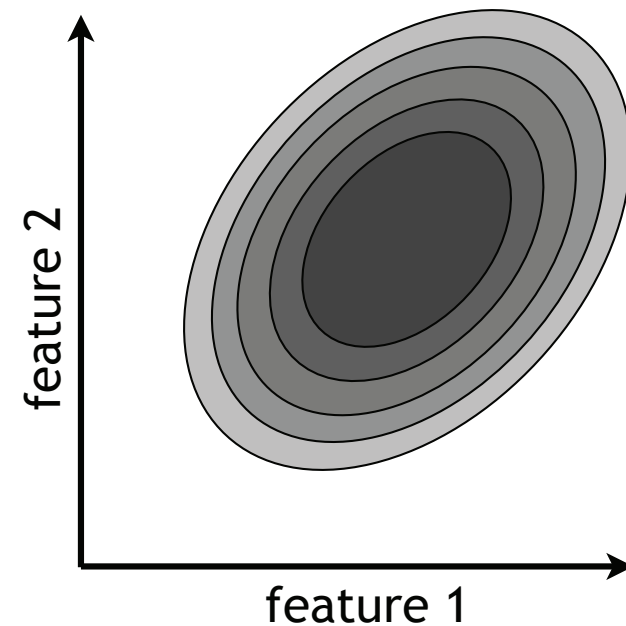


# DISCRIMINATIVE VS. GENERATIVE LEARNING

DISCRIMINATIVE  
*task-dependent representation*



GENERATIVE  
*task-independent representation*





# TWO TASKS



Neil  
Hounsby



Ferenc  
Huszár



Mohammad  
Ghassemi



Gergő  
Orbán



Daniel  
Wolpert

# TWO TASKS



Neil  
Hounsby



Ferenc  
Huszár



Mohammad  
Ghassemi

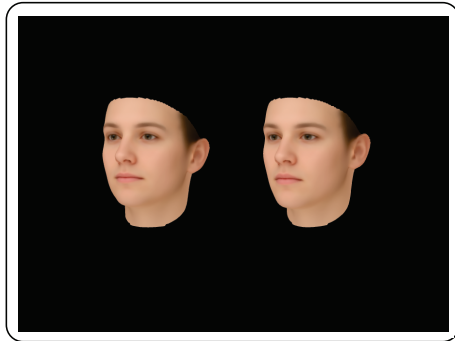


Gergő  
Orbán



Daniel  
Wolpert

**familiarity task**  
*“which face looks more familiar?”*



# TWO TASKS



Neil  
Hounsby



Ferenc  
Huszár



Mohammad  
Ghassemi

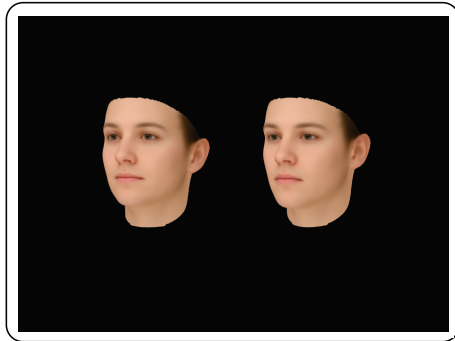


Gergő  
Orbán

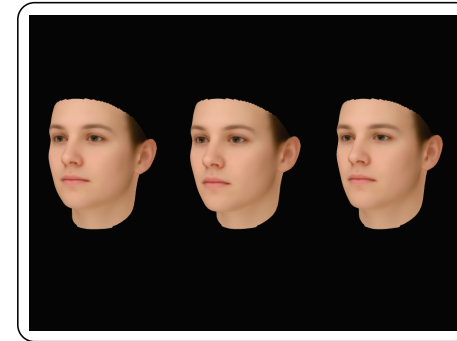


Daniel  
Wolpert

**familiarity task**  
*“which face looks more familiar?”*



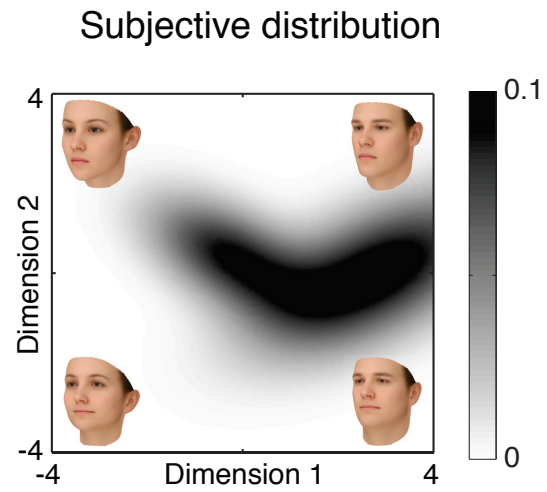
**odd-one-out task**  
*“which face is the odd-one-out?”*



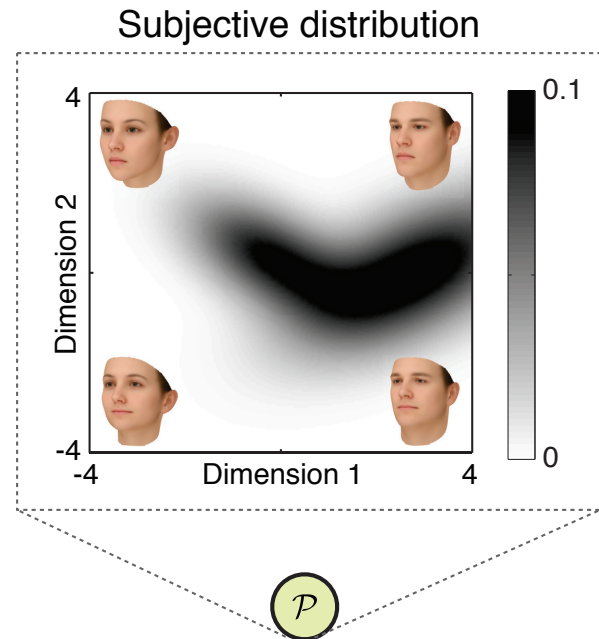
*Hounsby et al, Curr Biol 2013*

# COGNITIVE TOMOGRAPHY

# COGNITIVE TOMOGRAPHY

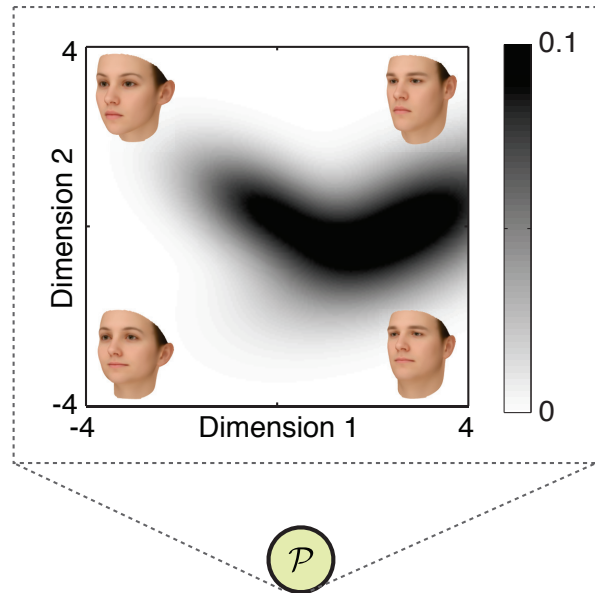


# COGNITIVE TOMOGRAPHY

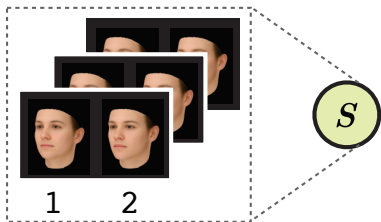


# COGNITIVE TOMOGRAPHY

Subjective distribution

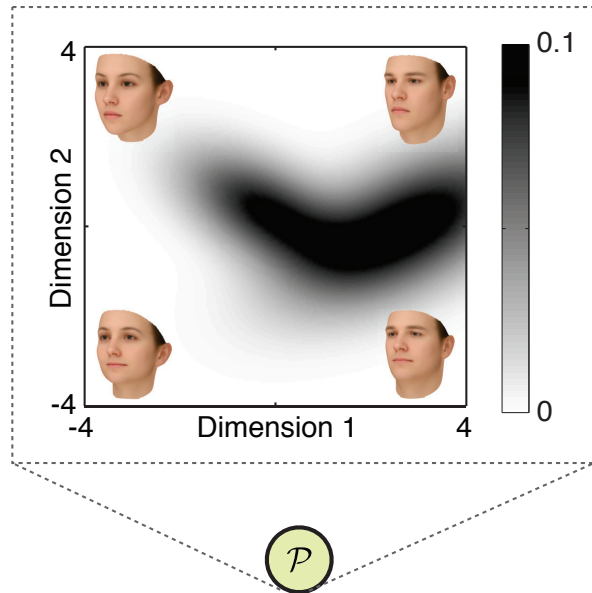


Stimuli

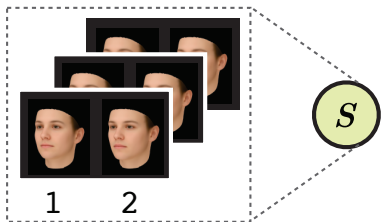


# COGNITIVE TOMOGRAPHY

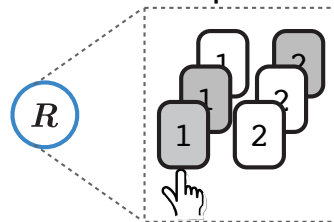
Subjective distribution



Stimuli



Responses

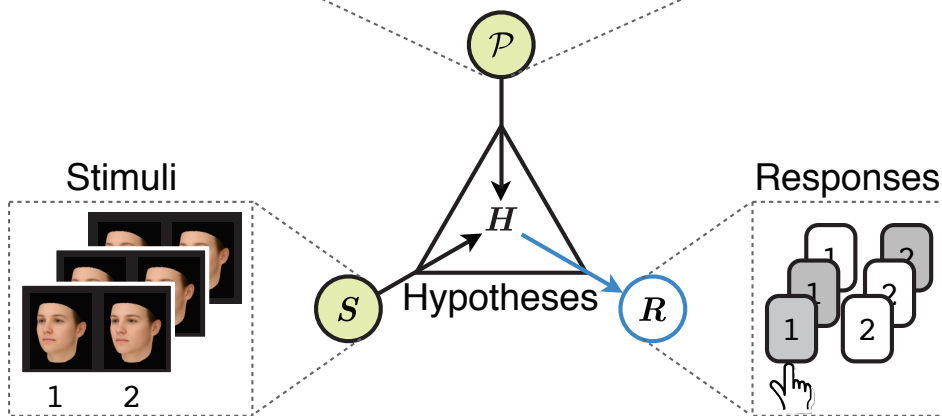
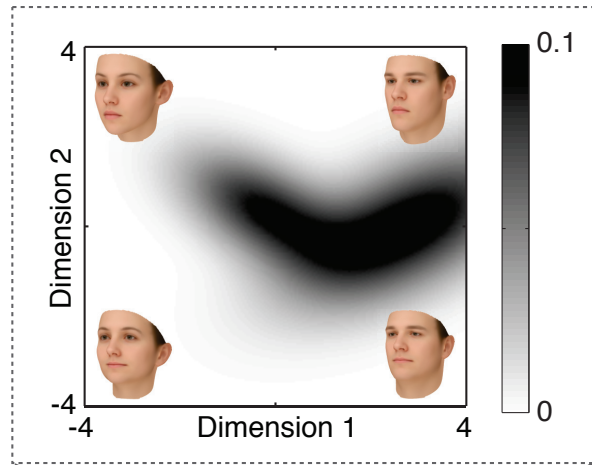




# COGNITIVE TOMOGRAPHY

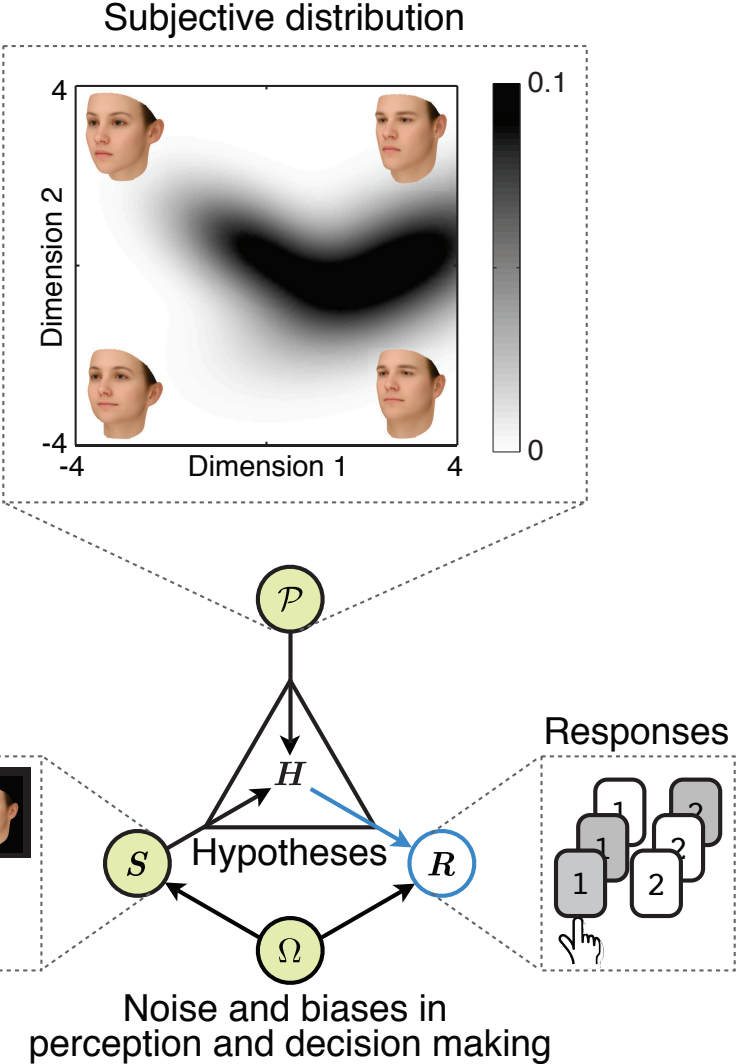
## ideal observer model

### Subjective distribution



# COGNITIVE TOMOGRAPHY

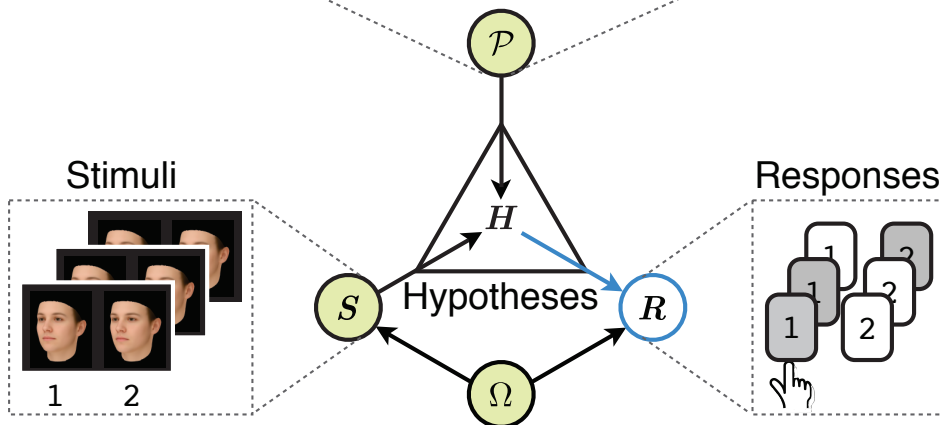
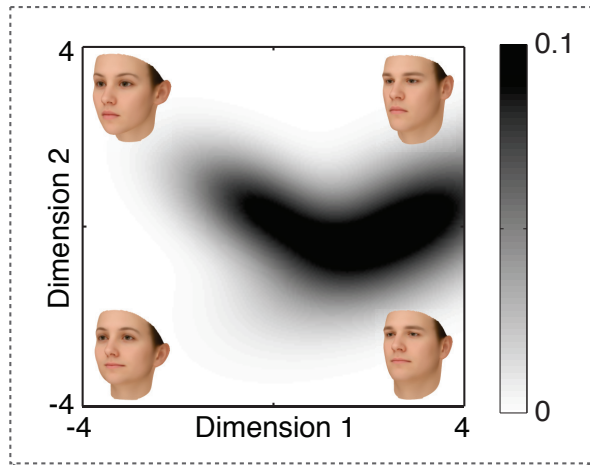
## quasi-ideal observer model



# COGNITIVE TOMOGRAPHY

## quasi-ideal observer model

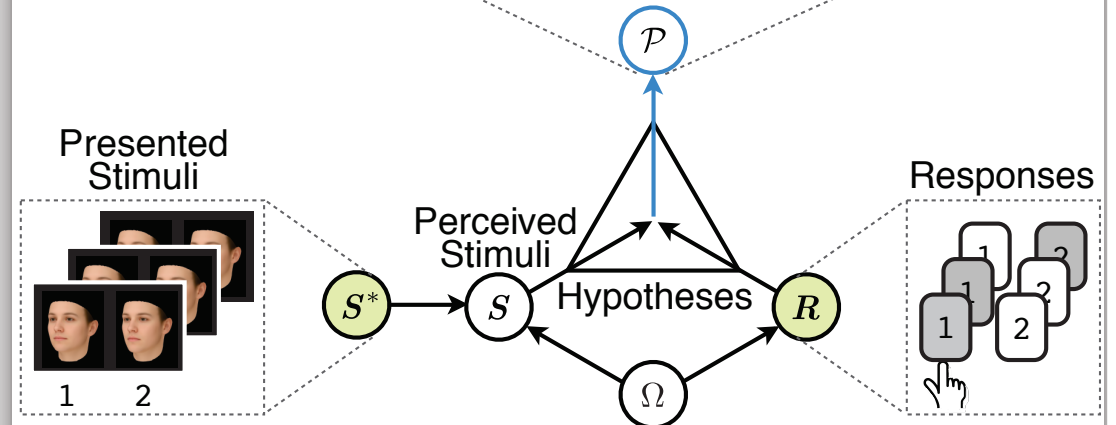
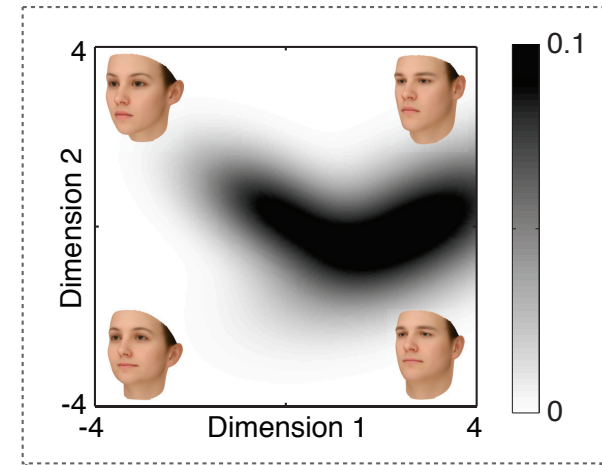
Subjective distribution



Noise and biases in perception and decision making

## cognitive tomography inverting the ideal observer model

Subjective distribution

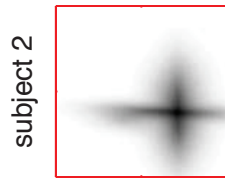


Noise and biases in perception and decision making

Houlsby et al, Curr Biol 2013

# SUBJECTIVE DISTRIBUTIONS

# SUBJECTIVE DISTRIBUTIONS



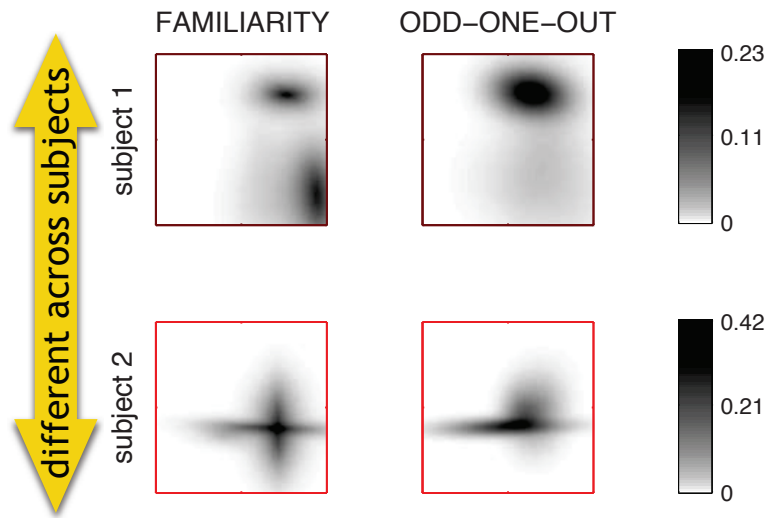
# SUBJECTIVE DISTRIBUTIONS



# SUBJECTIVE DISTRIBUTIONS

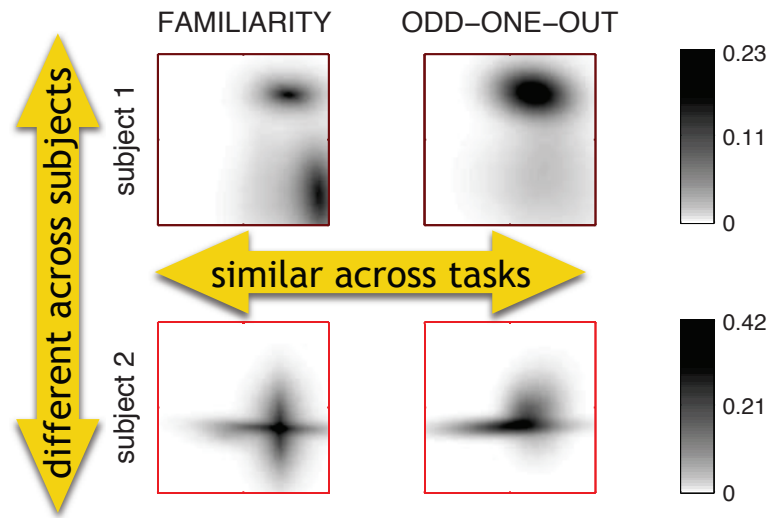


# SUBJECTIVE DISTRIBUTIONS

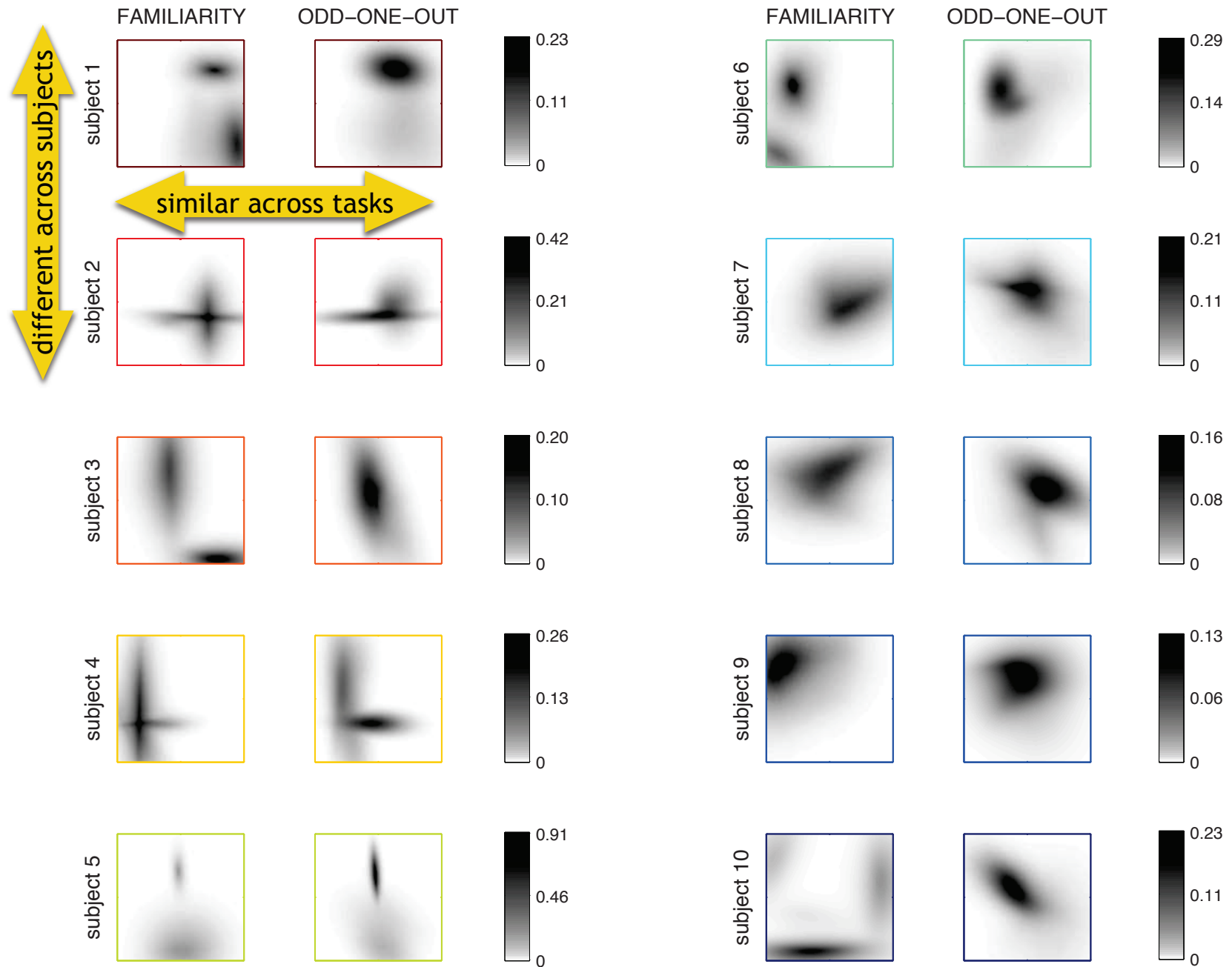




# SUBJECTIVE DISTRIBUTIONS



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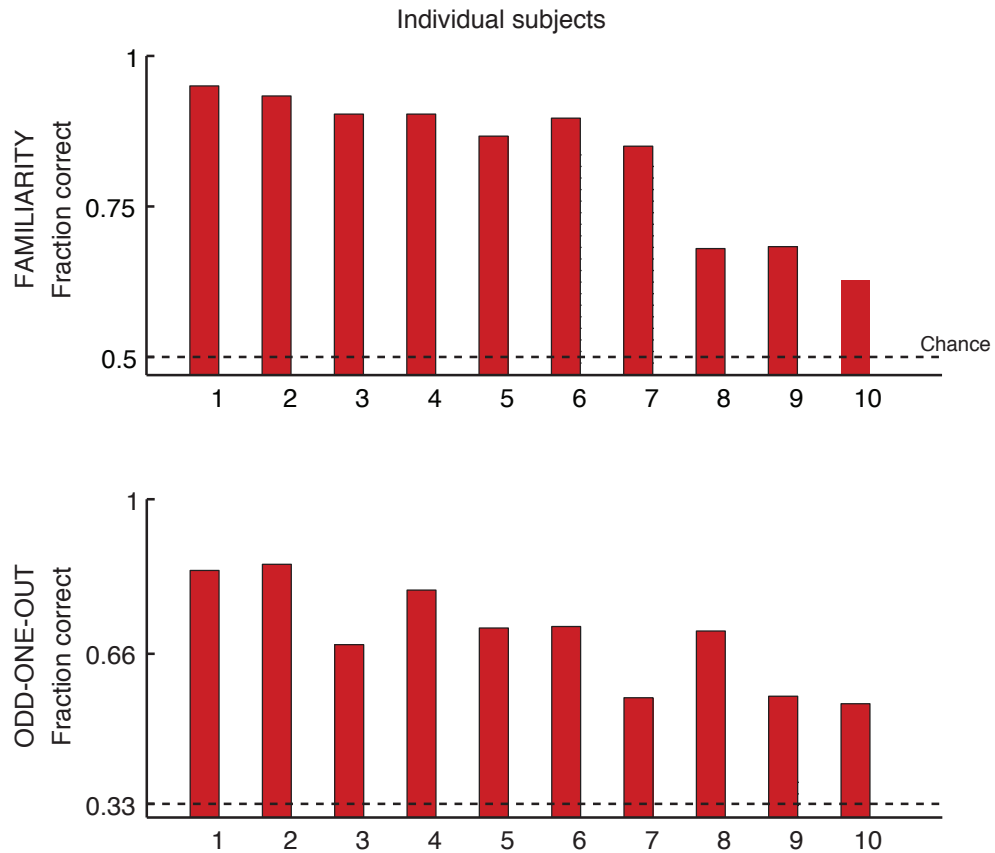


Houlsby et al, Curr Biol 2013

# PREDICTING BEHAVIOUR

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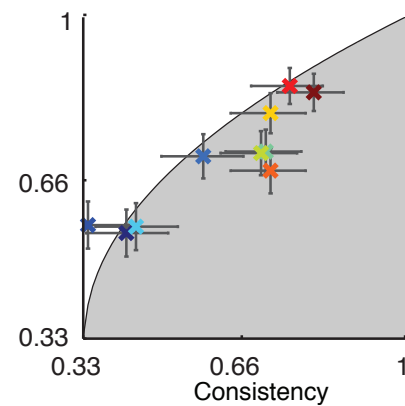
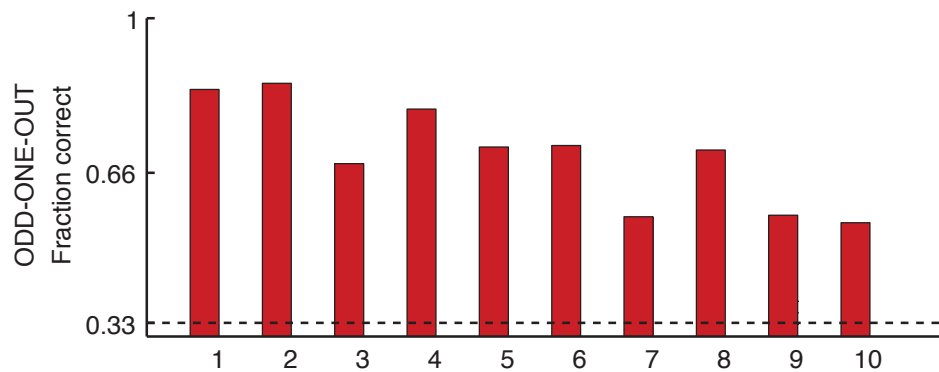
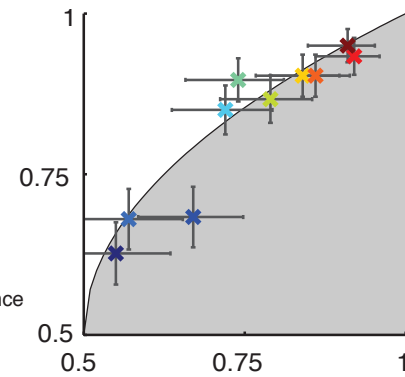
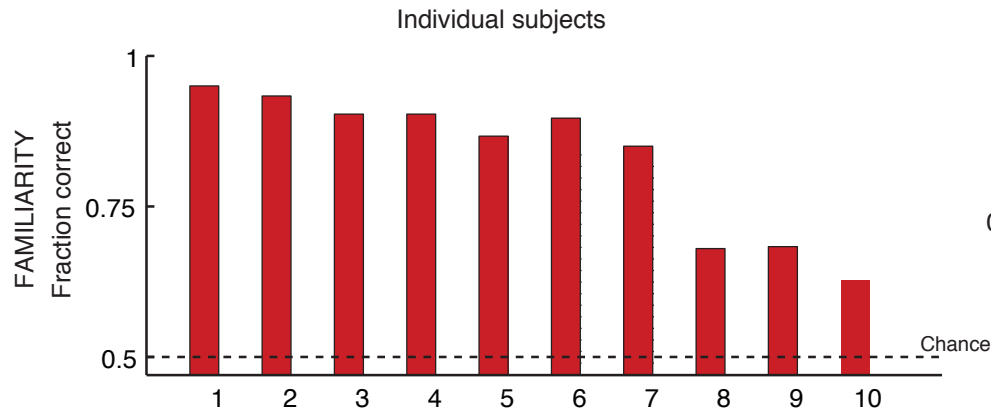
well above chance



# PREDICTING BEHAVIOUR

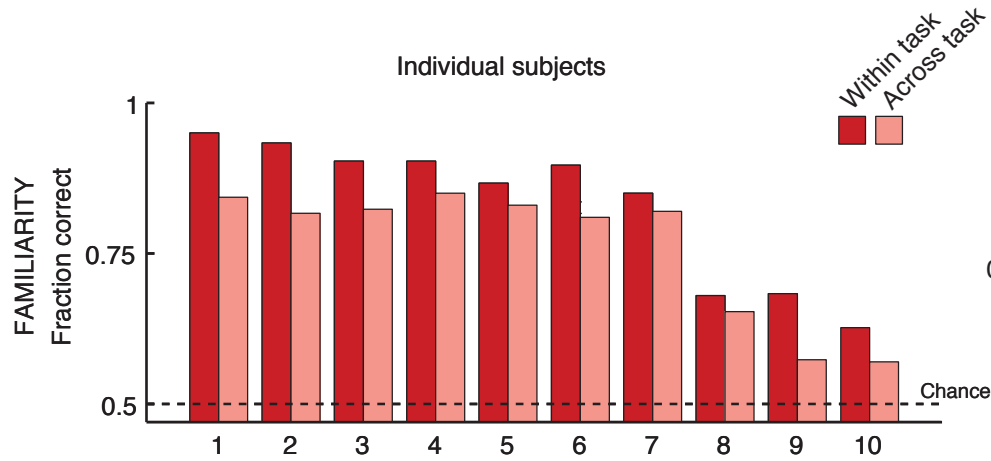
well above chance

close to theoretical upper bound

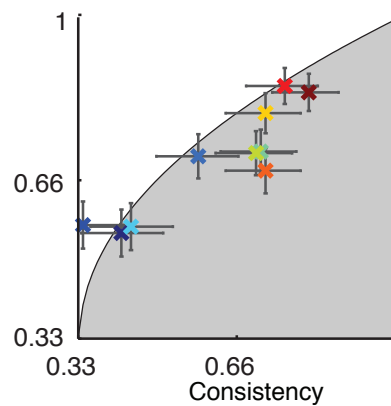
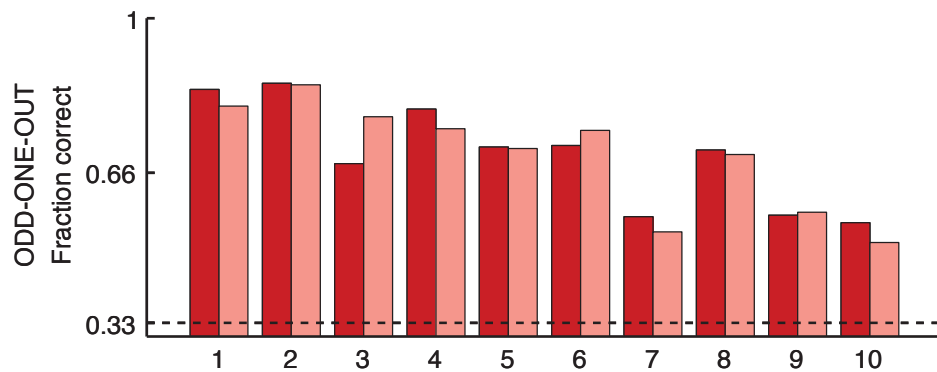
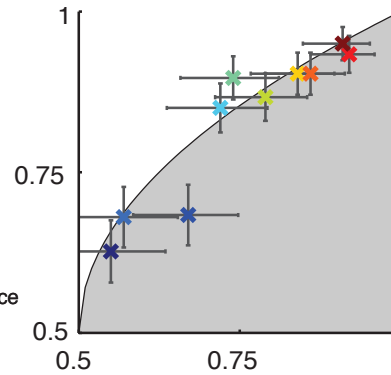


# PREDICTING BEHAVIOUR

well above chance  
both within and across tasks



close to theoretical  
upper bound

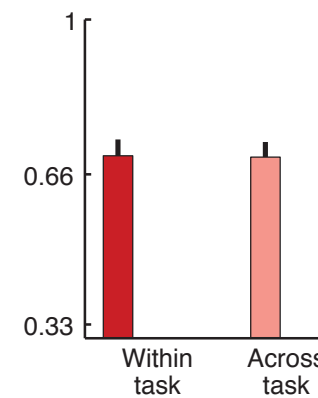
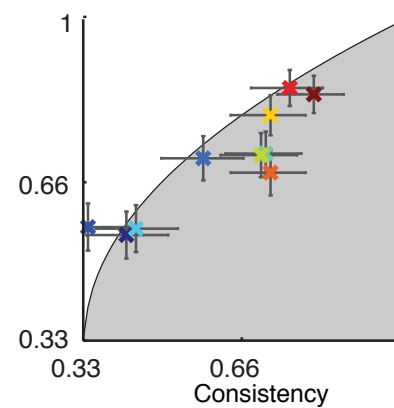
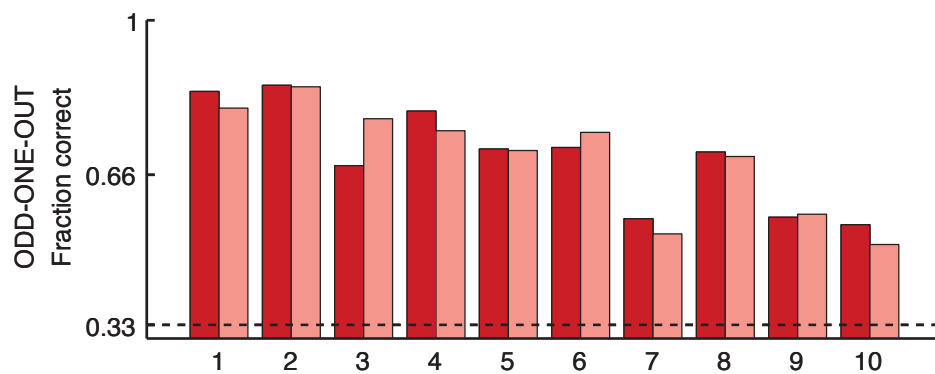
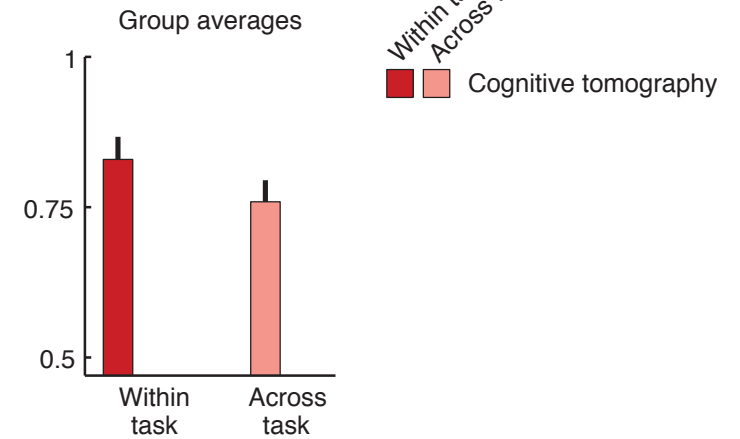
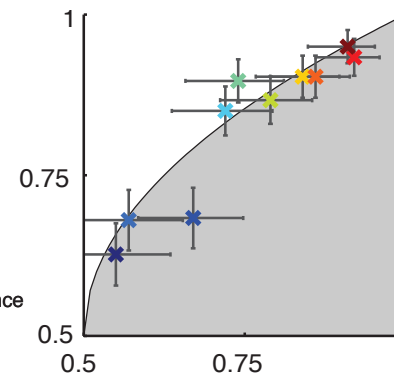
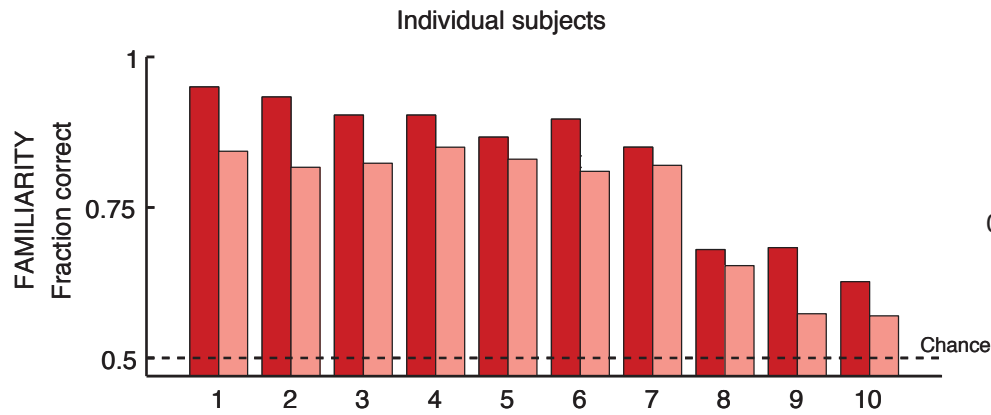


# PREDICTING BEHAVIOUR

well above chance  
both within and across tasks

close to theoretical  
upper bound

better than  
controls



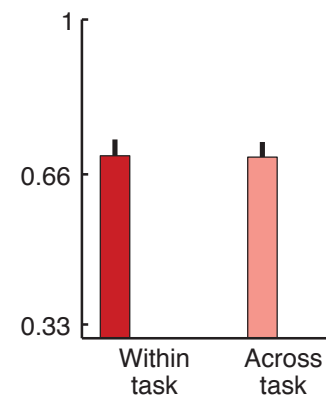
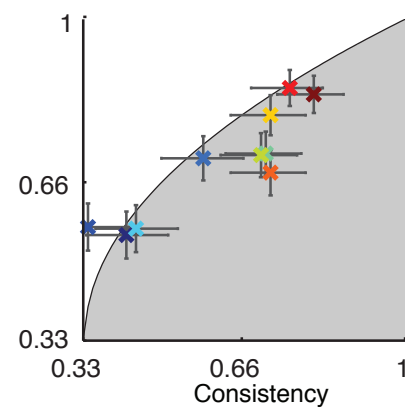
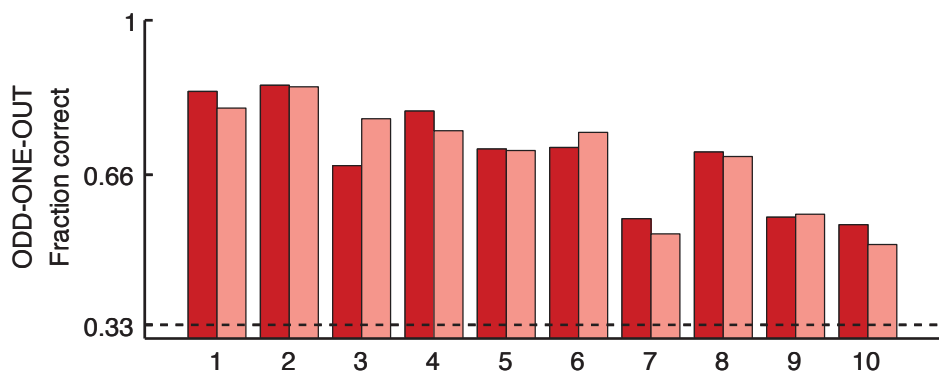
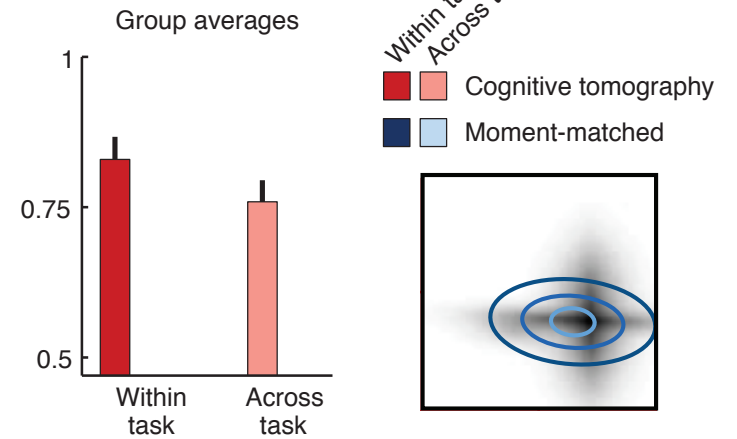
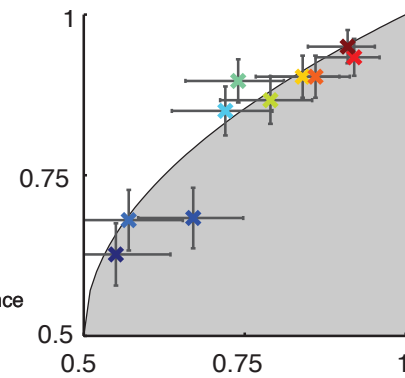
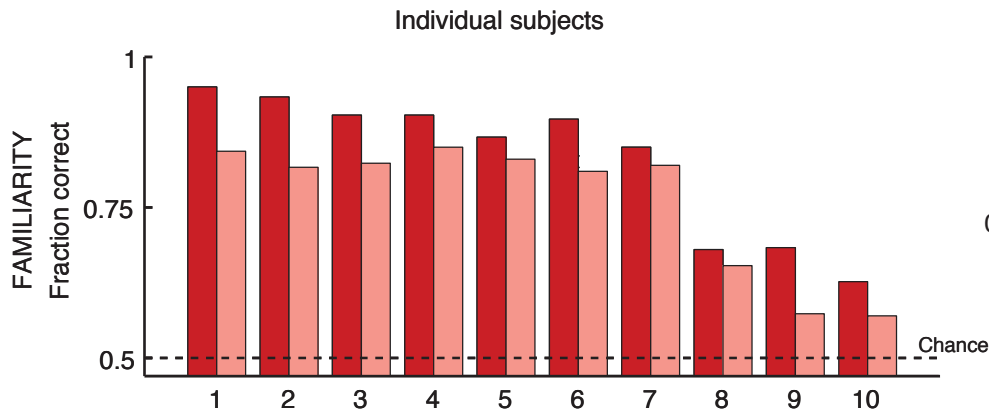
Houlsby et al, Curr Biol 2013

# PREDICTING BEHAVIOUR

well above chance  
both within and across tasks

close to theoretical  
upper bound

better than  
controls

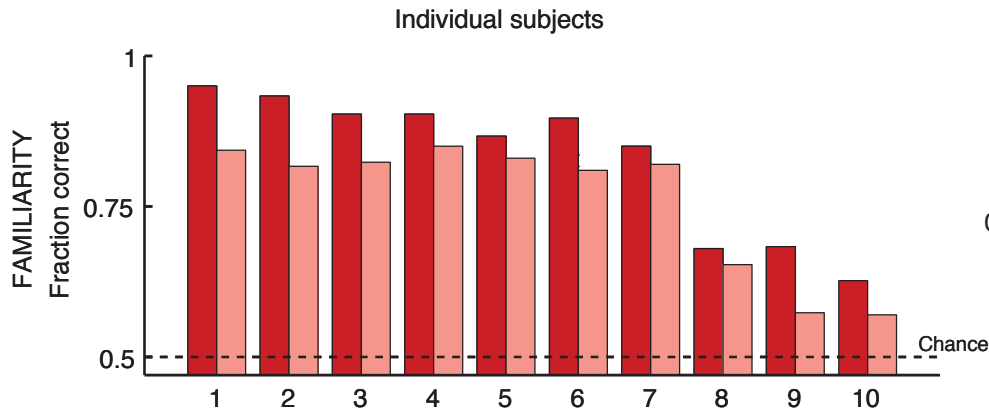


Houlsby et al, Curr Biol 2013

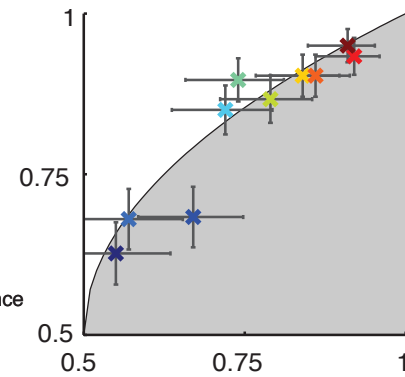


# PREDICTING BEHAVIOUR

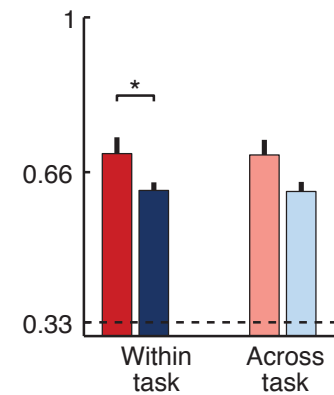
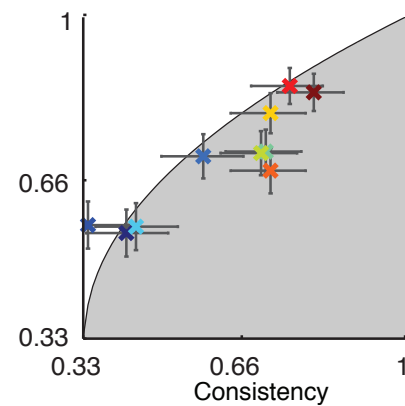
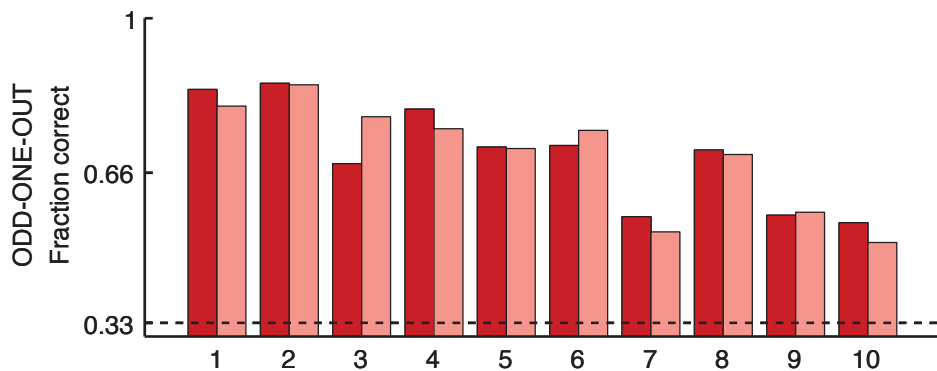
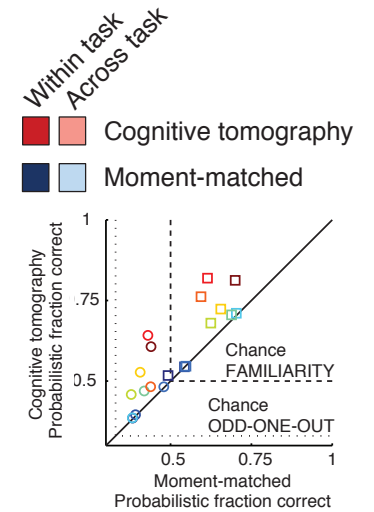
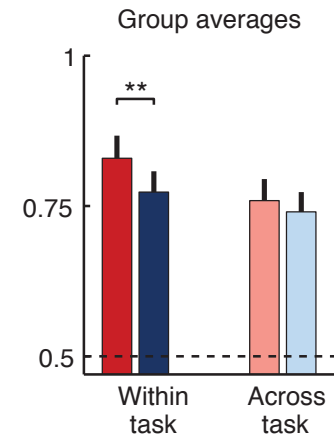
well above chance  
both within and across tasks



close to theoretical  
upper bound



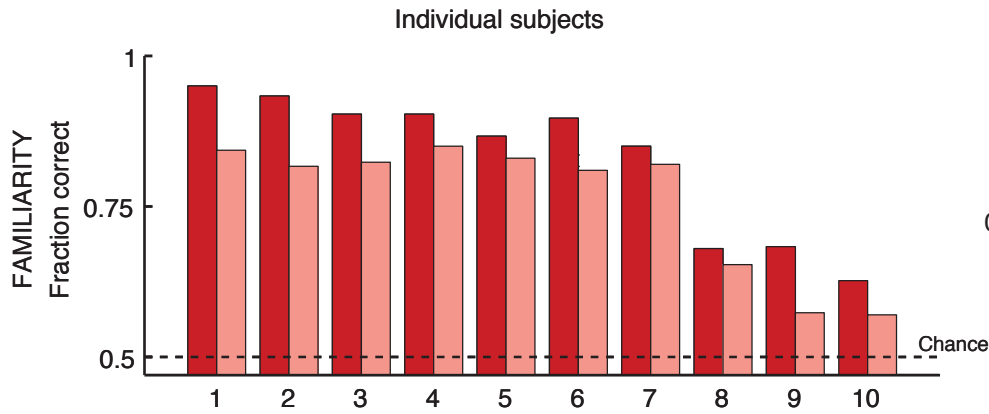
better than  
controls



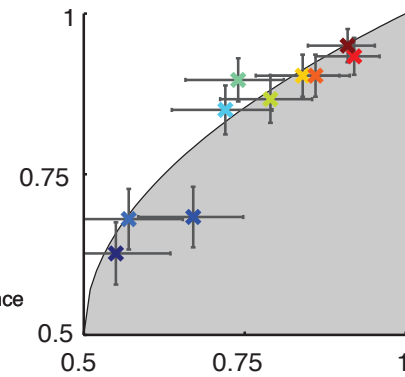
Houlsby et al, Curr Biol 2013

# PREDICTING BEHAVIOUR

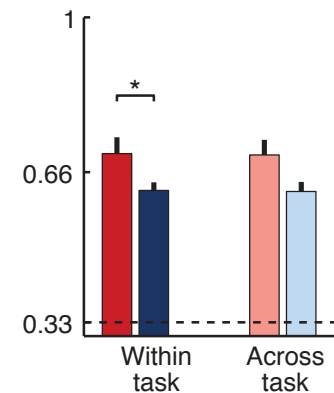
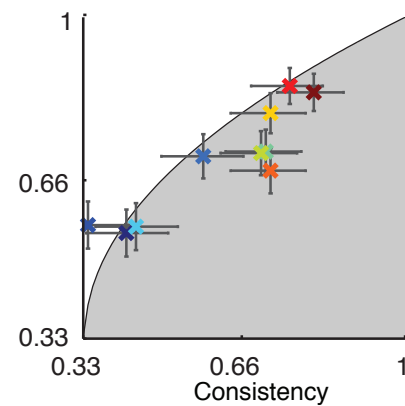
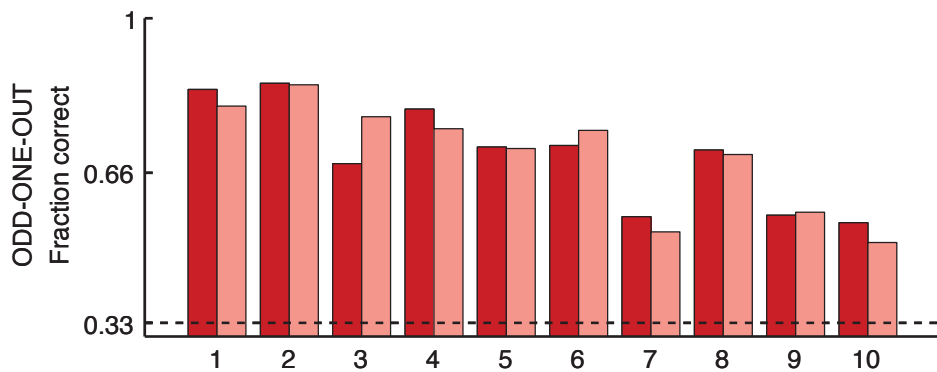
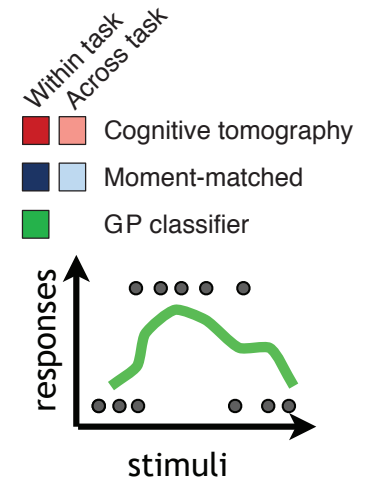
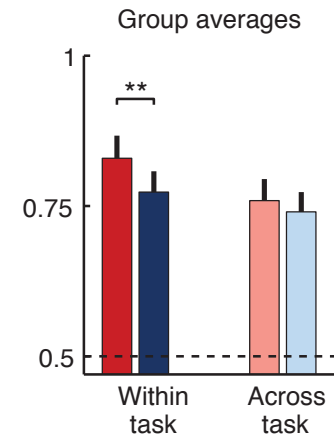
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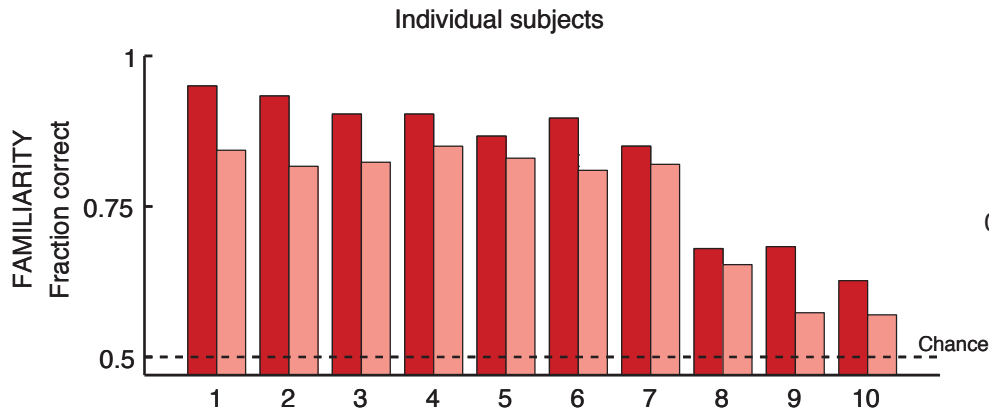
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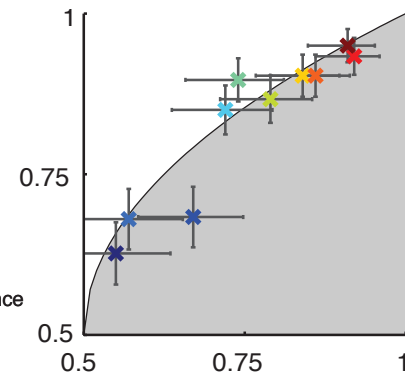
Houlsby et al, Curr Biol 2013

# PREDICTING BEHAVIOUR

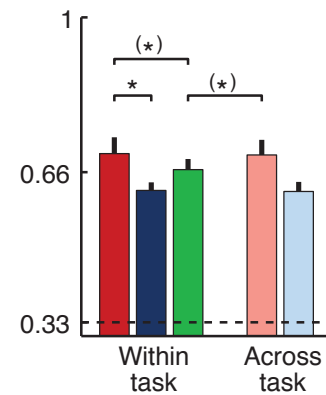
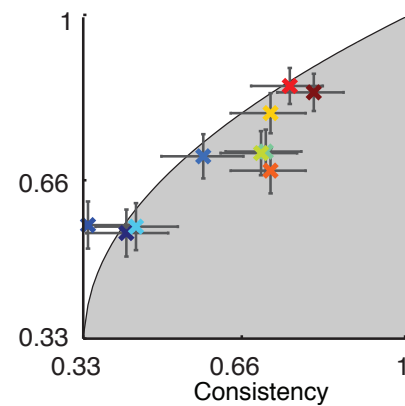
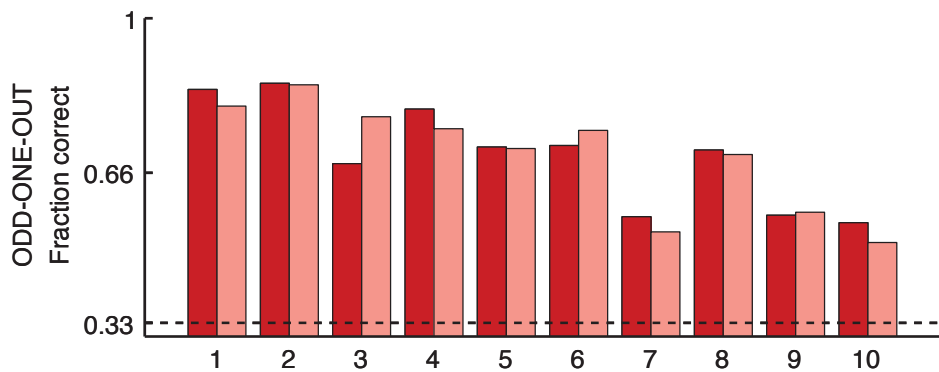
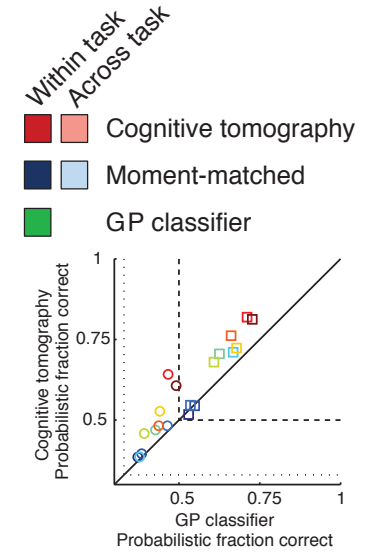
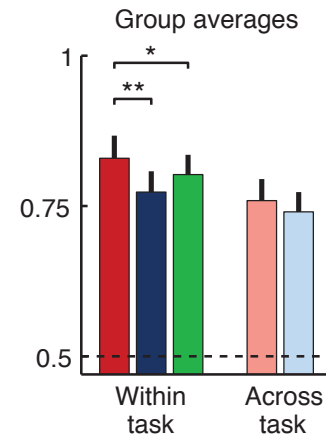
well above chance  
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controls



Houlsby et al, Curr Biol 2013

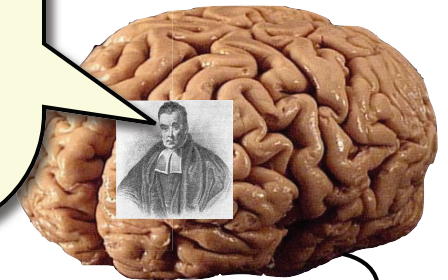
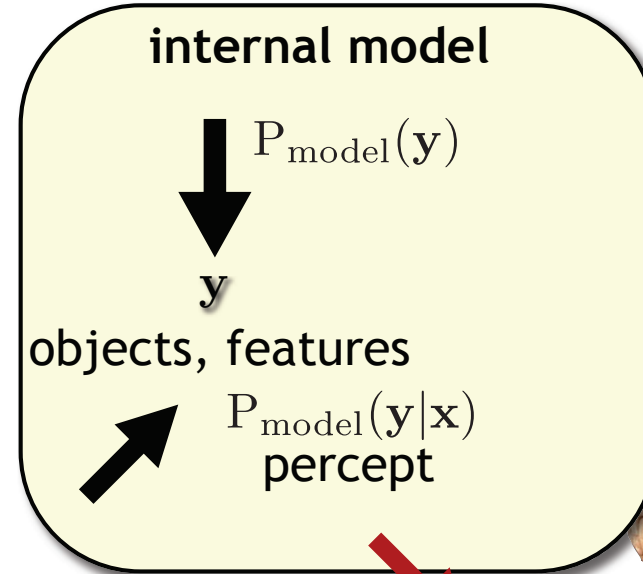
# PROBABILISTIC INTERNAL MODELS



$P_{\text{true}}(\mathbf{y})$   
↓  
 $\mathbf{y}$   
objects, features

$P_{\text{true}}(\mathbf{x}|\mathbf{y})$   
↘

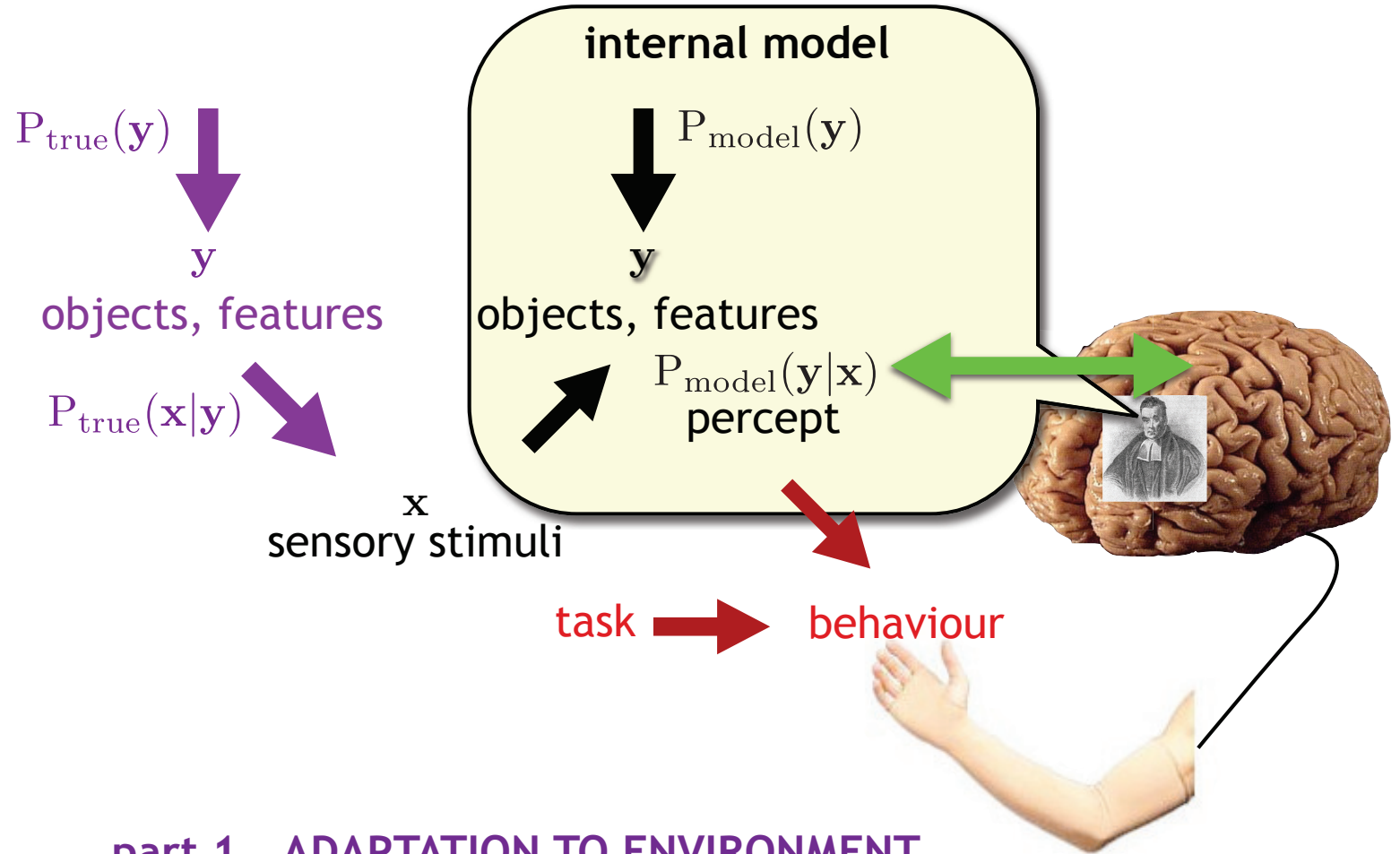
$\mathbf{x}$   
sensory stimuli



part 1. ADAPTATION TO ENVIRONMENT

part 2. TASK-INDEPENDENCE

# PROBABILISTIC INTERNAL MODELS



part 1. ADAPTATION TO ENVIRONMENT

part 2. TASK-INDEPENDENCE

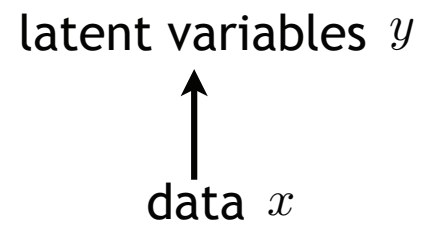
part 3. NEURAL SUBSTRATES

# BAYESIAN INFERENCE IN THE BRAIN

# BAYESIAN INFERENCE IN THE BRAIN

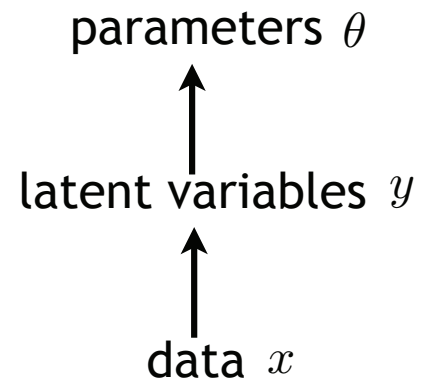
data  $x$

# BAYESIAN INFERENCE IN THE BRAIN

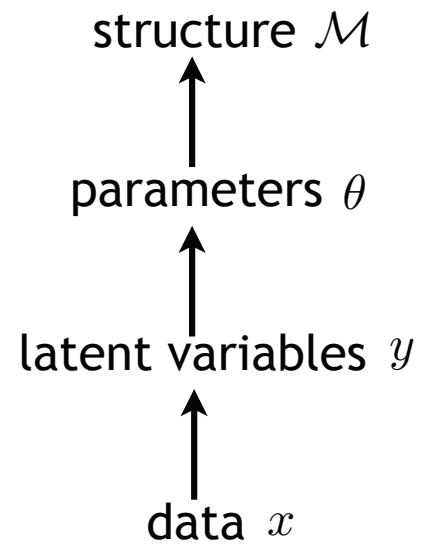




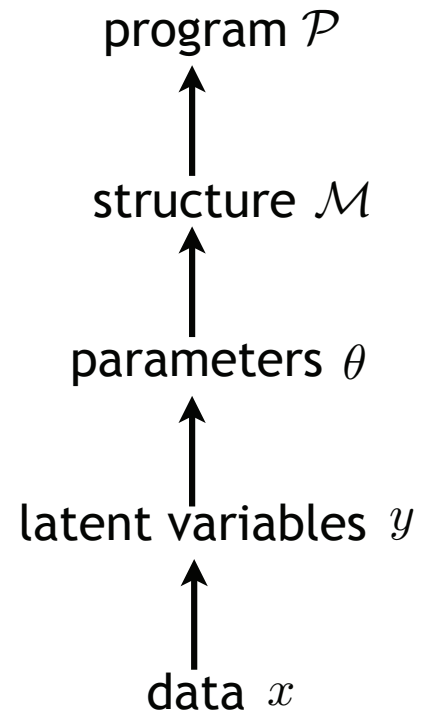
# BAYESIAN INFERENCE IN THE BRAIN



# BAYESIAN INFERENCE IN THE BRAIN

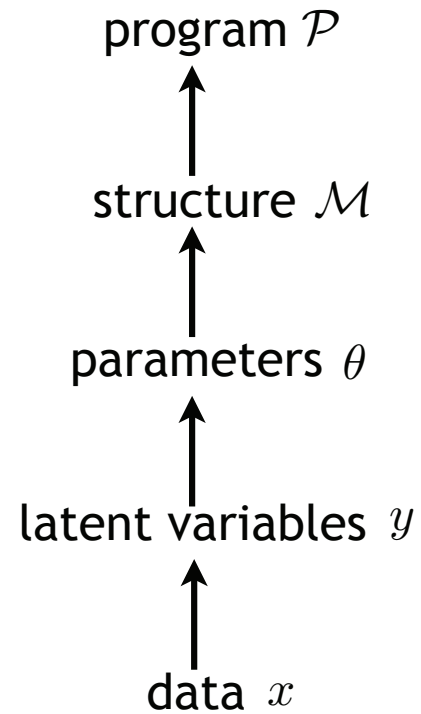


# BAYESIAN INFERENCE IN THE BRAIN



# BAYESIAN INFERENCE IN THE BRAIN

cognitive science



# BAYESIAN INFERENCE IN THE BRAIN

cognitive science

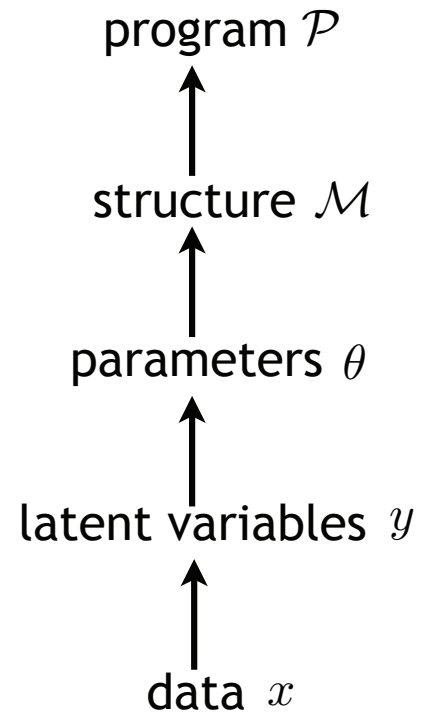
*theory*

✓

✓

✓

✓



# BAYESIAN INFERENCE IN THE BRAIN

cognitive science

*theory*

*experiments*

✓

✓

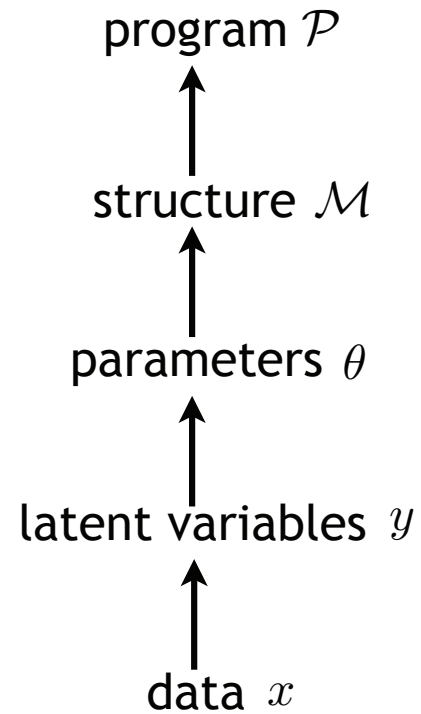
✓

✓

✓

✓

✓



# BAYESIAN INFERENCE IN THE BRAIN

cognitive science

*theory*

*experiments*

✓

?

✓

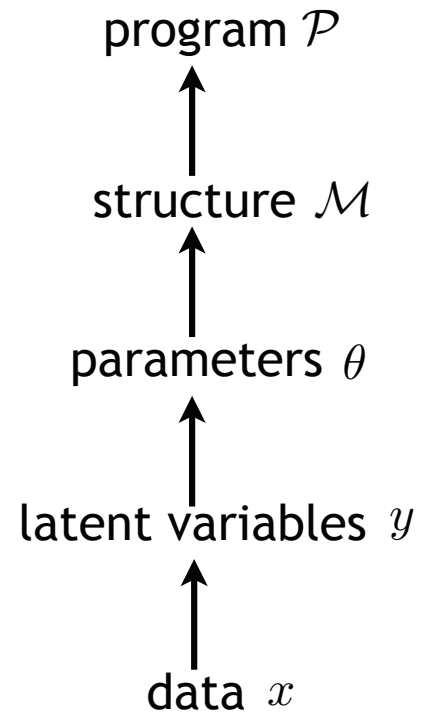
✓

✓

✓

✓

✓



# BAYESIAN INFERENCE IN THE BRAIN

cognitive science

neuroscience

*theory*

*experiments*

✓

?

✓

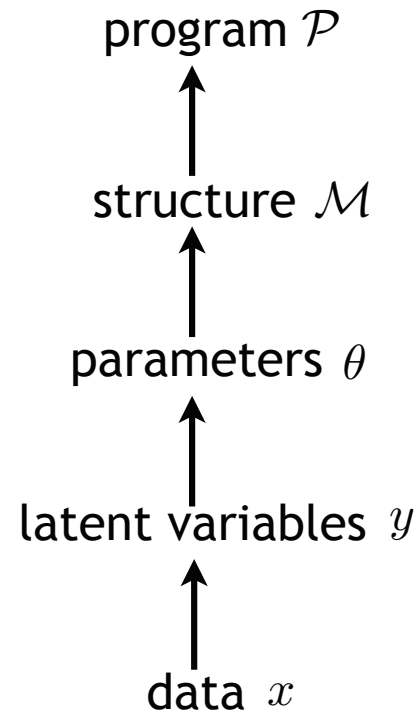
✓

✓

✓

✓

✓





# BAYESIAN INFERENCE IN THE BRAIN

cognitive science

neuroscience

*theory*

*experiments*

*theory*

✓

?

✓

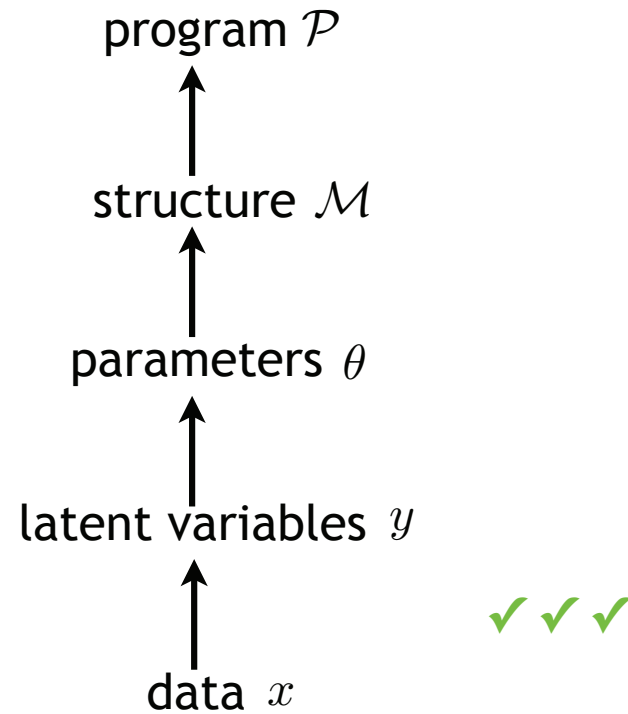
✓

✓

✓

✓

✓



# BAYESIAN INFERENCE IN THE BRAIN

cognitive science

neuroscience

*theory*

*experiments*

*theory*

✓

?

✓

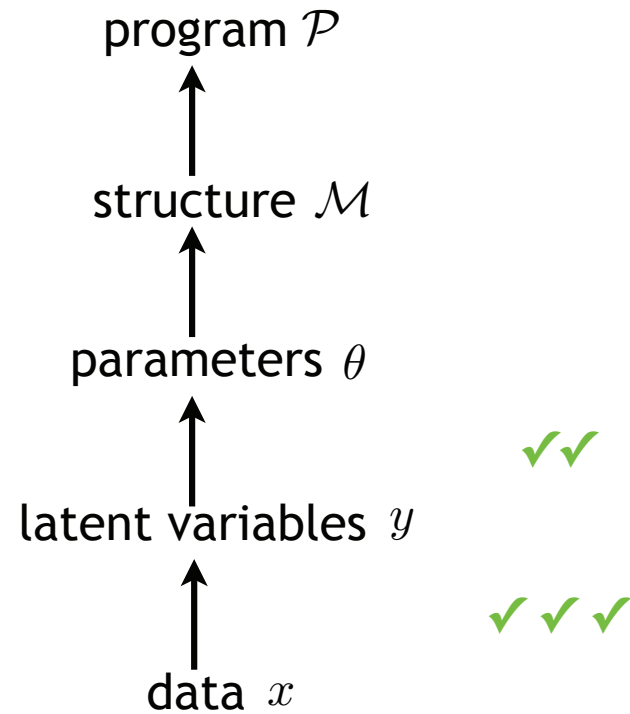
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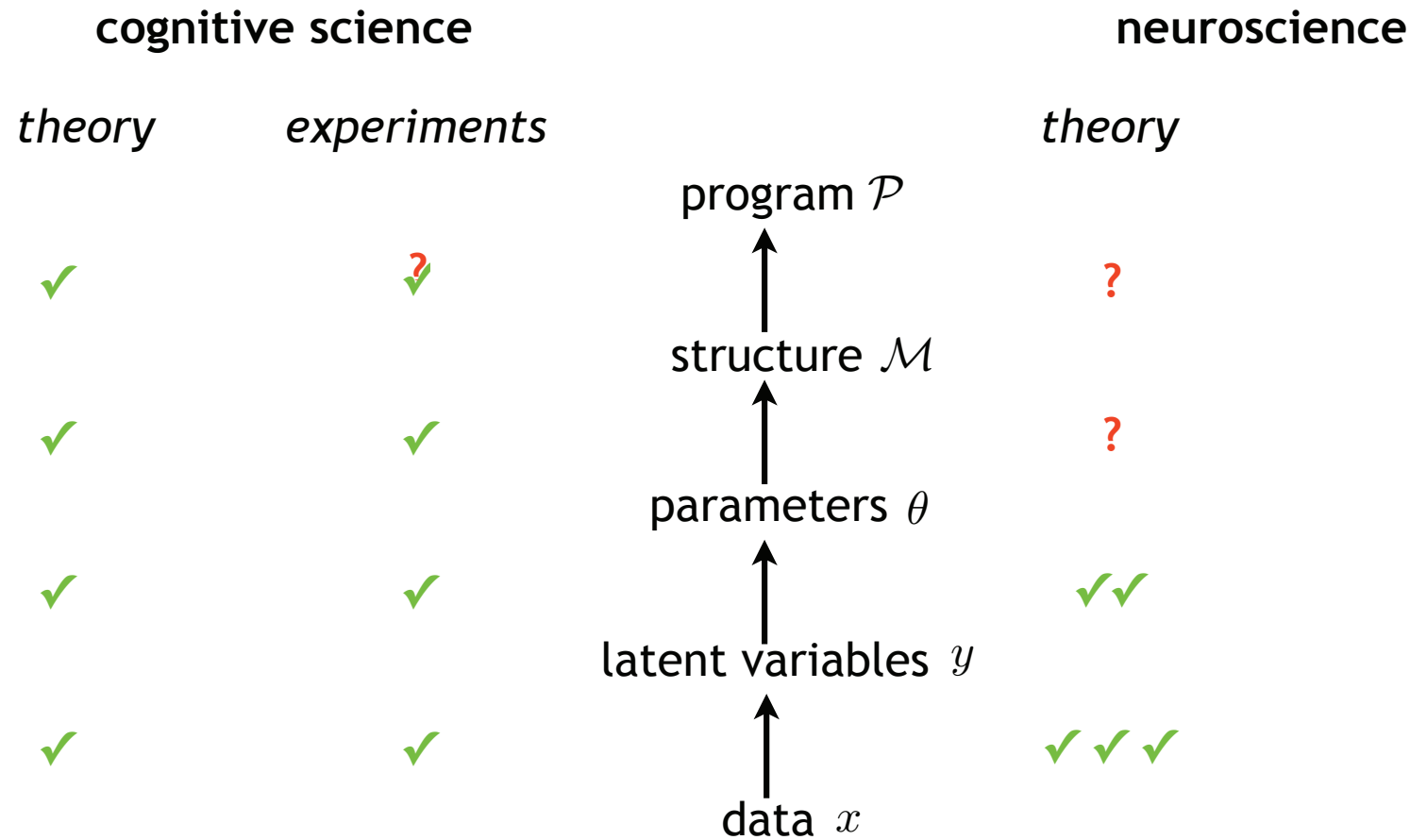
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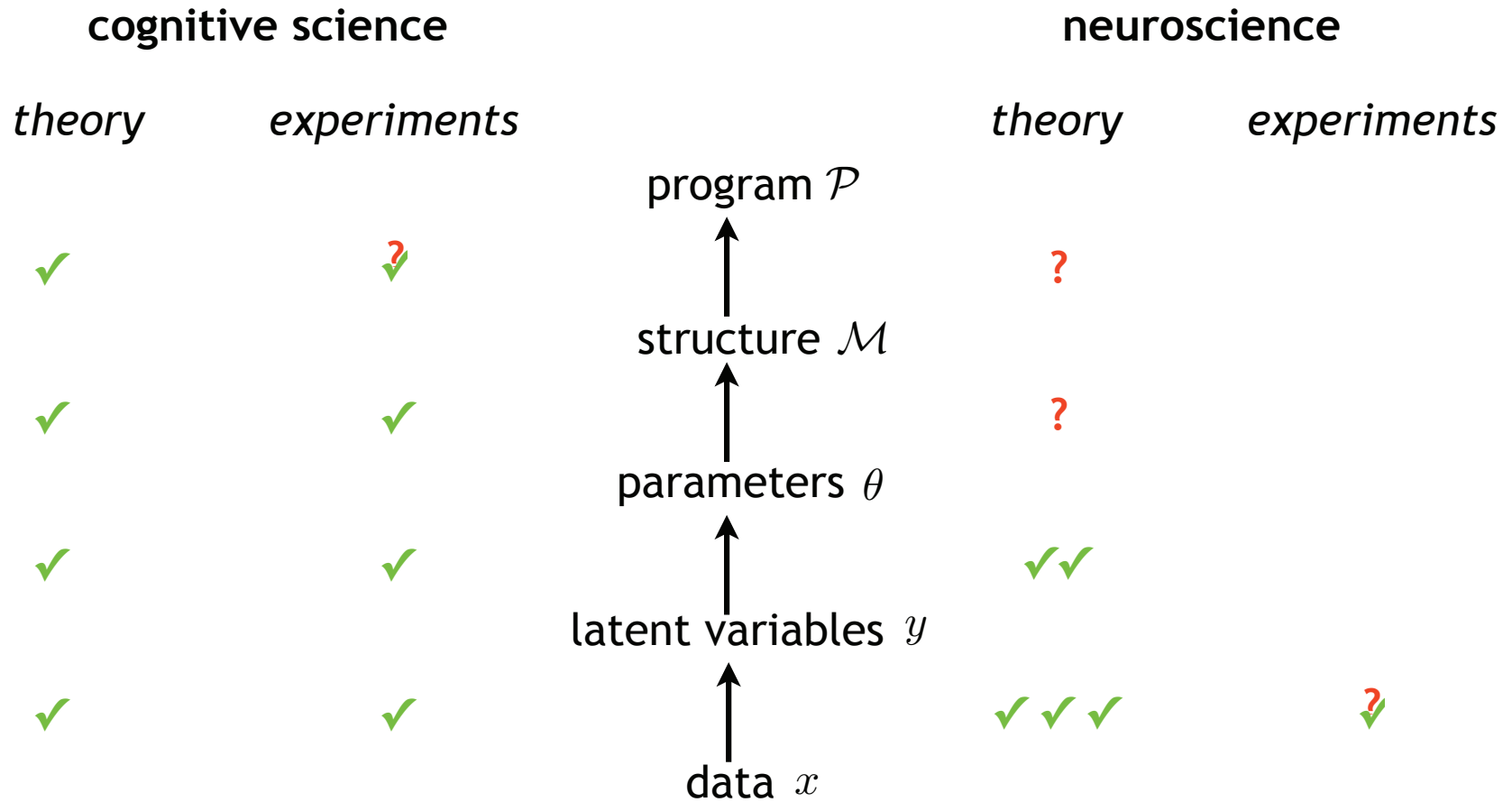
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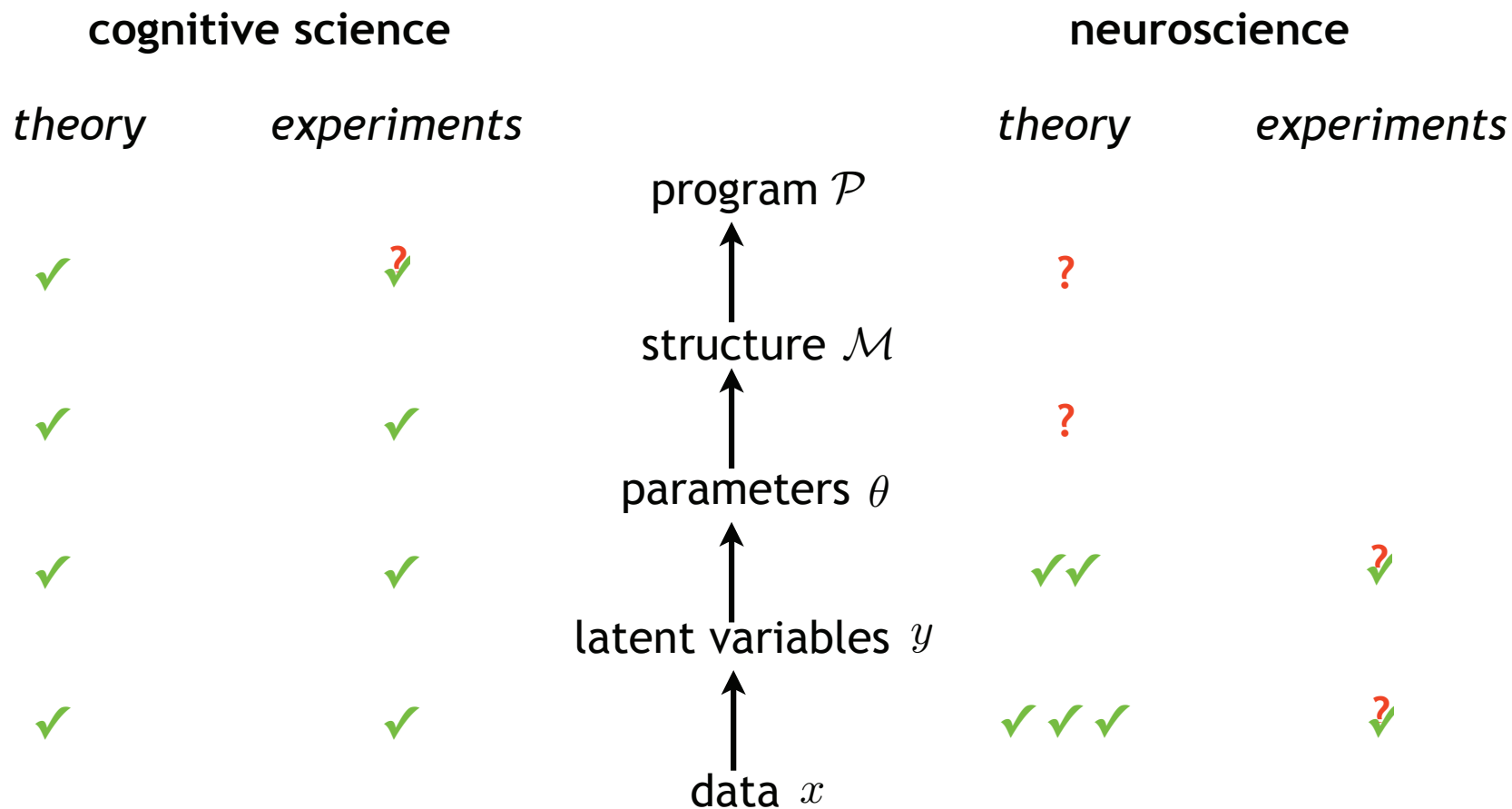
# BAYESIAN INFERENCE IN THE BRAIN



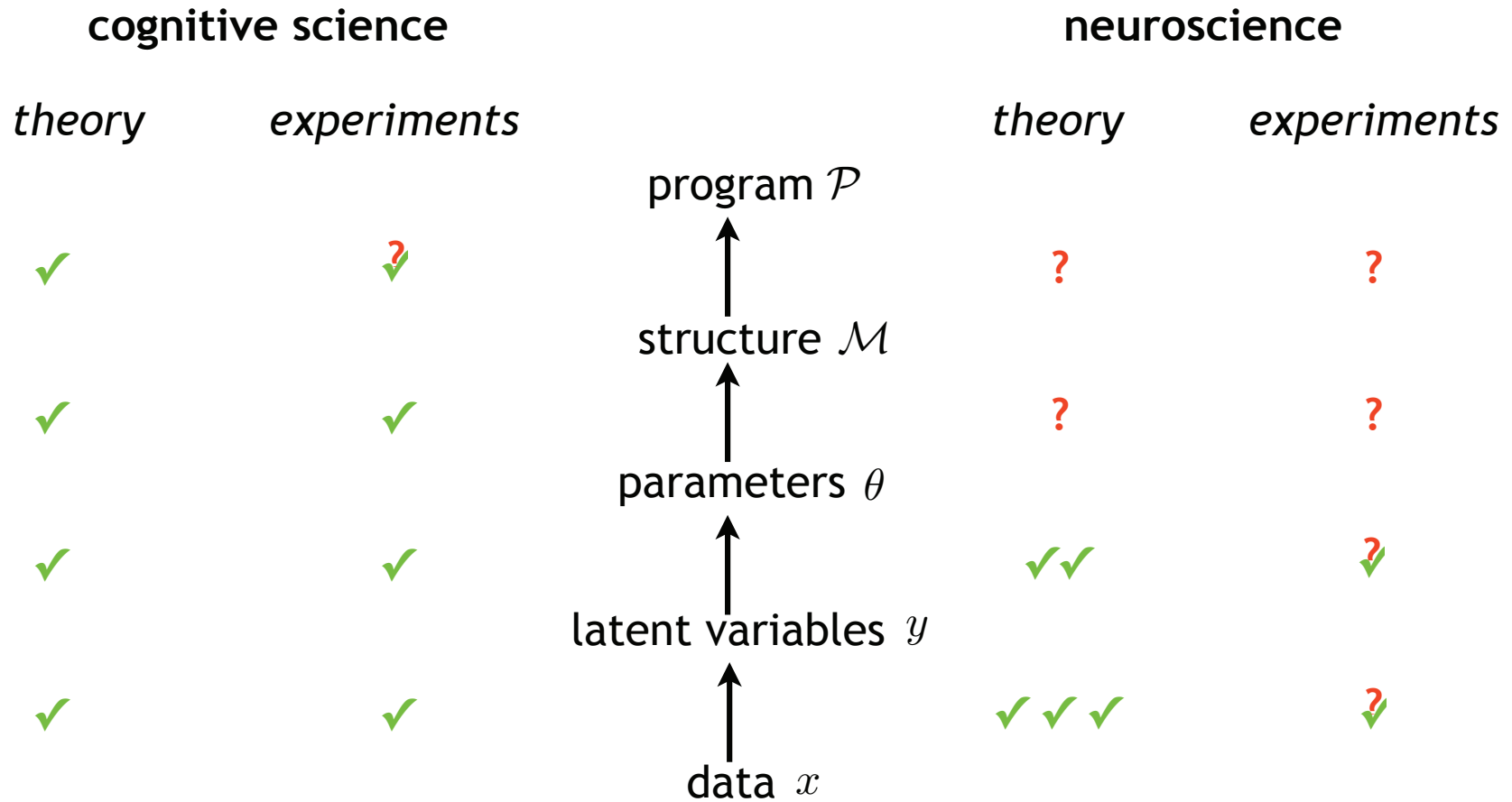
# BAYESIAN INFERENCE IN THE BRAIN



# BAYESIAN INFERENCE IN THE BRAIN



# BAYESIAN INFERENCE IN THE BRAIN



# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

# PERCEPTUAL UNCERTAINTY $\leftrightarrow$ NEURAL VARIABILITY VIA SAMPLING



# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY VIA SAMPLING



stimulus

# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY VIA SAMPLING



stimulus

# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY VIA SAMPLING



stimulus

# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY VIA SAMPLING

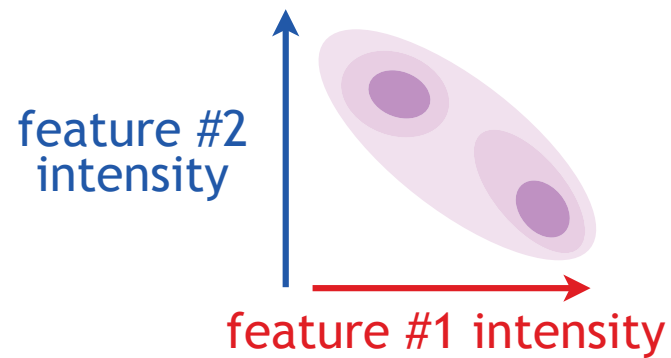
*ideal observer*

VIA SAMPLING



stimulus

$$P(\text{feature 1, feature 2} \mid \text{stimulus})$$



# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

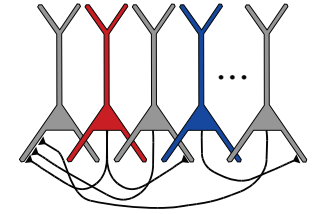
*ideal observer*

VIA SAMPLING

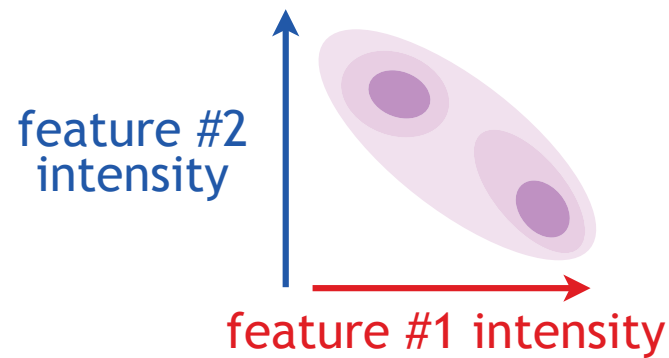
*visual cortex*



stimulus



$$P(\text{feature 1, feature 2} \mid \text{stimulus})$$



# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

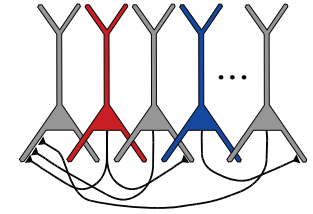
*ideal observer*

VIA SAMPLING

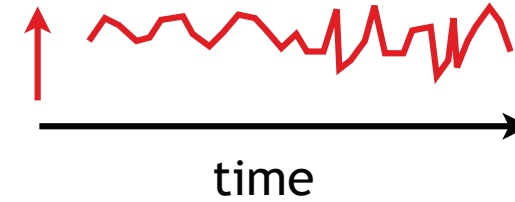
*visual cortex*



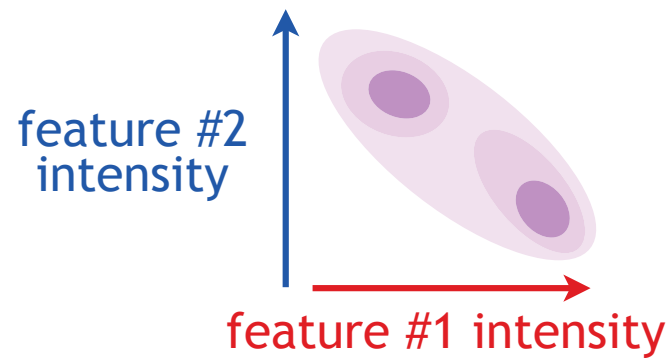
stimulus



cell #1  
response



$$P(\text{feature 1, feature 2} \mid \text{stimulus})$$



# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

*ideal observer*

VIA SAMPLING

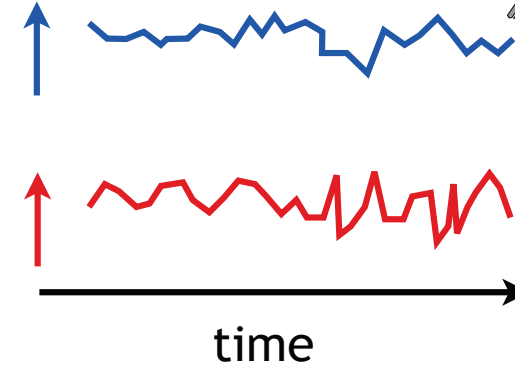
*visual cortex*



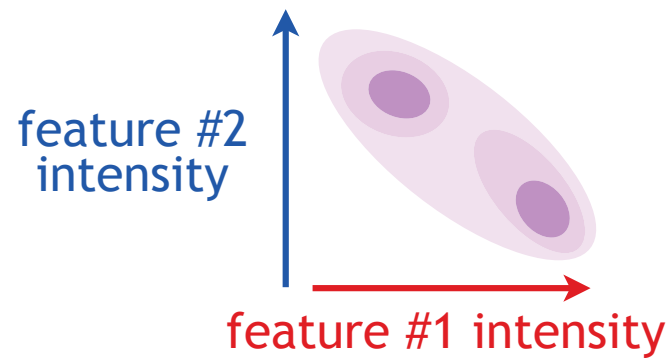
stimulus

cell #2  
response

cell #1  
response



$$P(\text{feature 1, feature 2} \mid \text{stimulus})$$

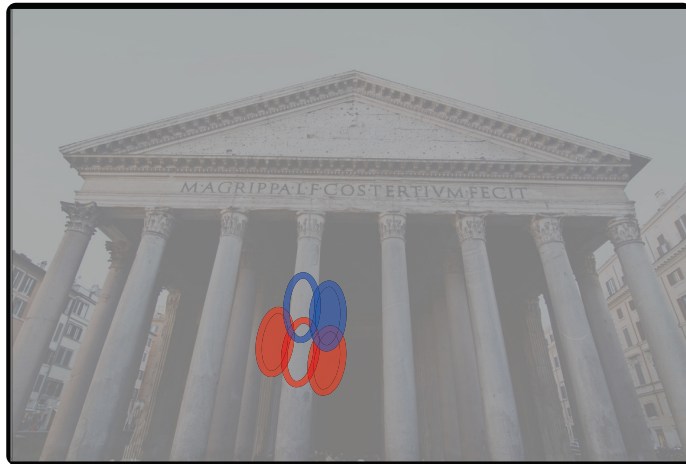


# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

*ideal observer*

VIA SAMPLING

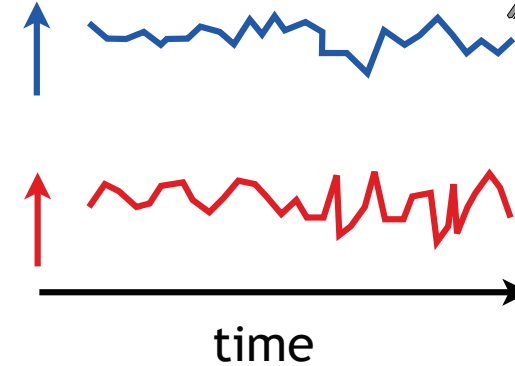
*visual cortex*



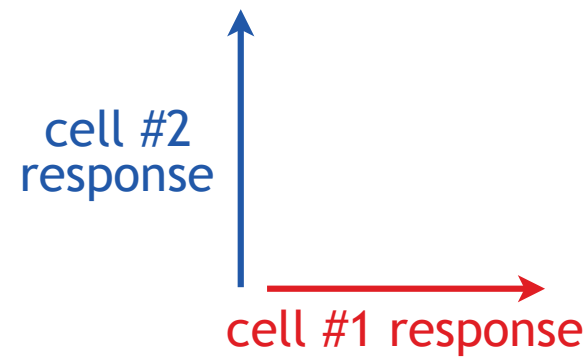
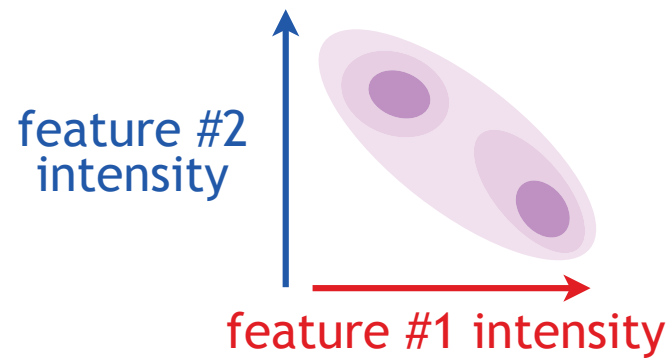
stimulus

cell #2 response

cell #1 response



$$P(\text{feature 1, feature 2} \mid \text{stimulus})$$





# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

*ideal observer*

VIA SAMPLING

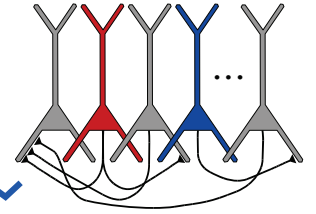
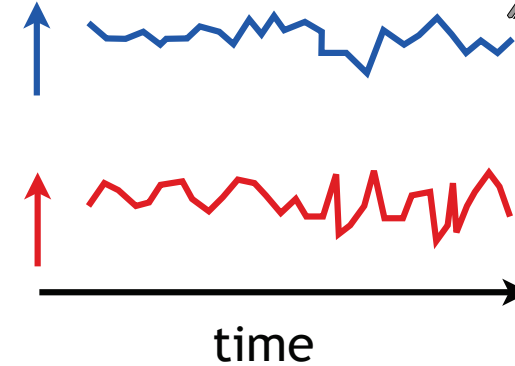
*visual cortex*



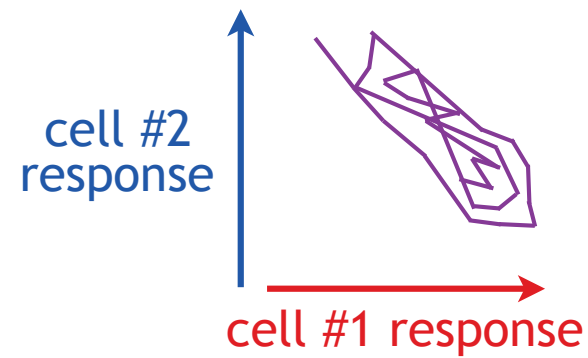
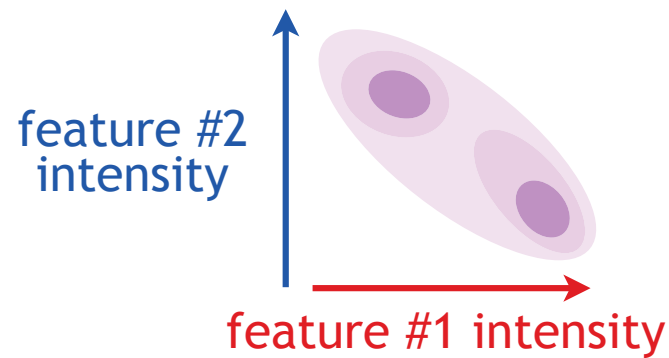
stimulus

cell #2  
response

cell #1  
response



$$P(\text{feature 1, feature 2} \mid \text{stimulus})$$



# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

*ideal observer*

VIA SAMPLING

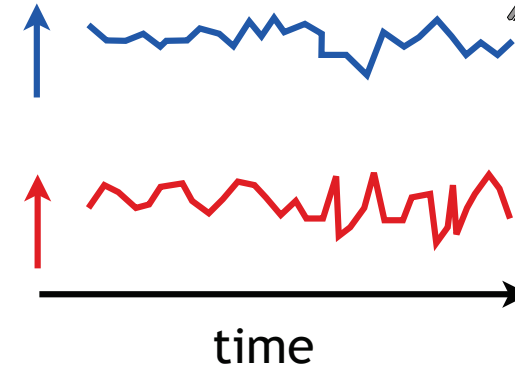
*visual cortex*



stimulus

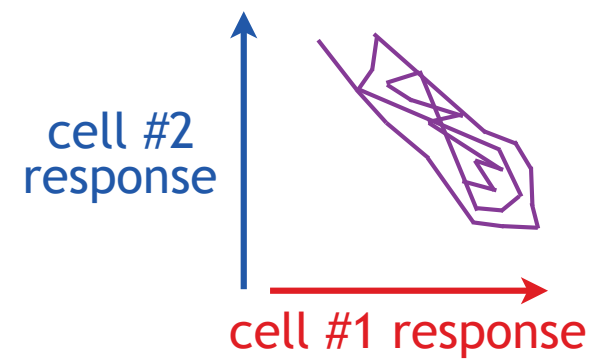
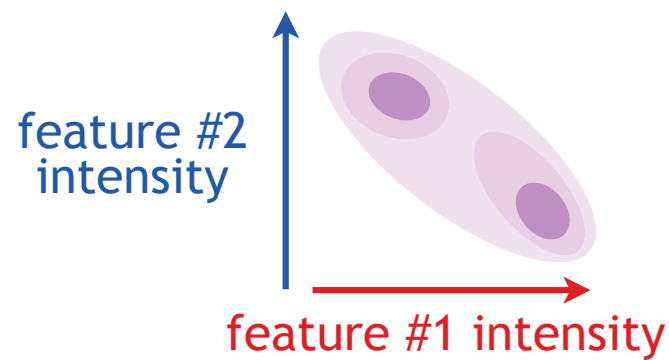
cell #2 response

cell #1 response



$$P(\text{feature \#1}, \text{feature \#2} \mid \text{stimulus})$$

$$P(\text{response \#1}, \text{response \#2} \mid \text{stimulus})$$



# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

ideal observer

VIA SAMPLING

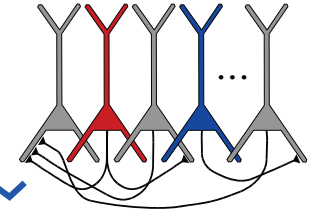
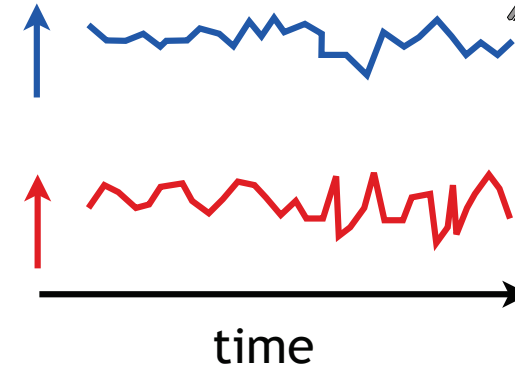
visual cortex



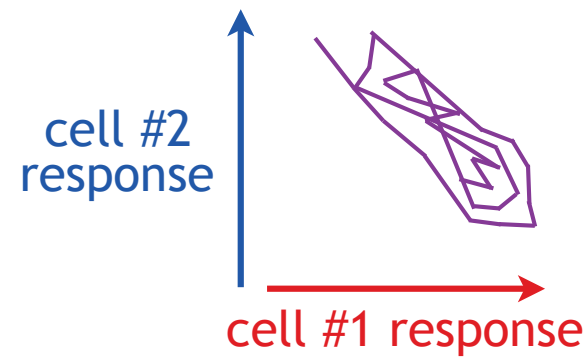
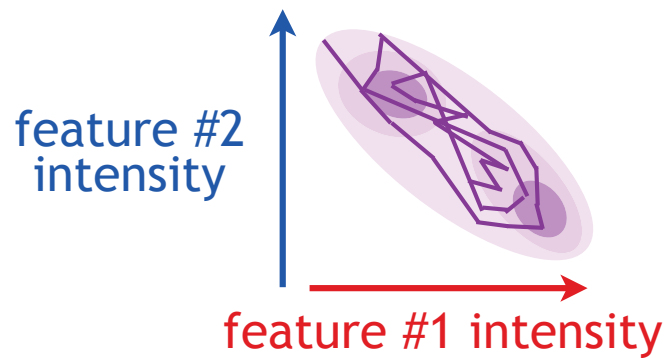
stimulus

cell #2 response

cell #1 response



$$P(\text{feature 1, feature 2} \mid \text{stimulus}) = P(\text{response 1, response 2} \mid \text{stimulus})$$



Fiser et al, TICS 2010

see also:

Hinton & Sejnowski, PDP 1986; Hinton et al, Science 1995; Dayan 1999; Hoyer & Hyvarinen, NIPS 2003, Lee & Mumford 2003

# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

ideal observer

VIA SAMPLING

visual cortex



stimulus

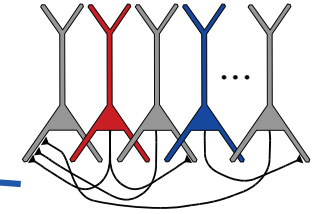
cell #2 response



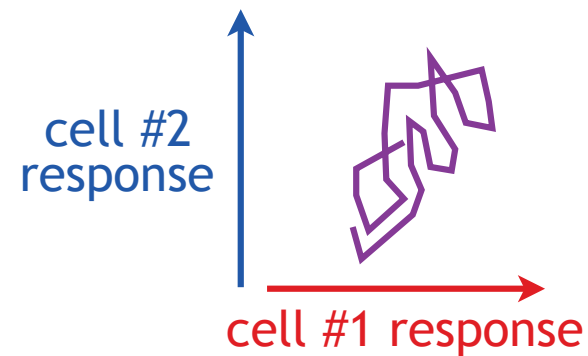
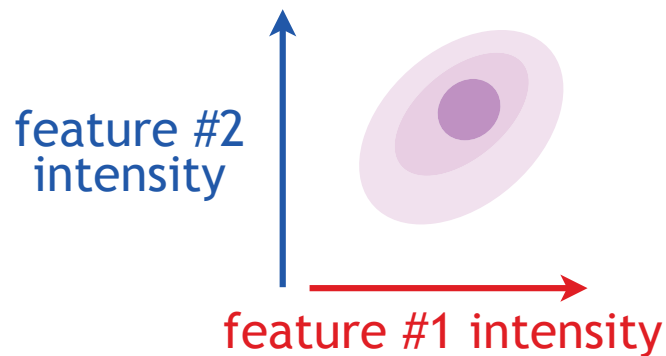
cell #1 response



time



$$P(\text{feature 1, feature 2} \mid \text{stimulus}) = P(\text{response 1, response 2} \mid \text{stimulus})$$



Fiser et al, TICS 2010

see also:

Hinton & Sejnowski, PDP 1986; Hinton et al, Science 1995; Dayan 1999; Hoyer & Hyvarinen, NIPS 2003, Lee & Mumford 2003

# PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

ideal observer

VIA SAMPLING

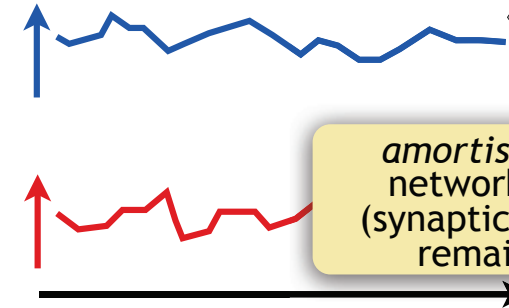
visual cortex



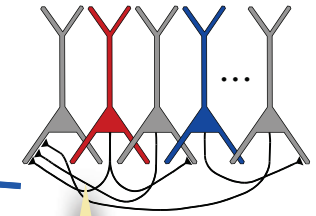
stimulus

cell #2 response

cell #1 response

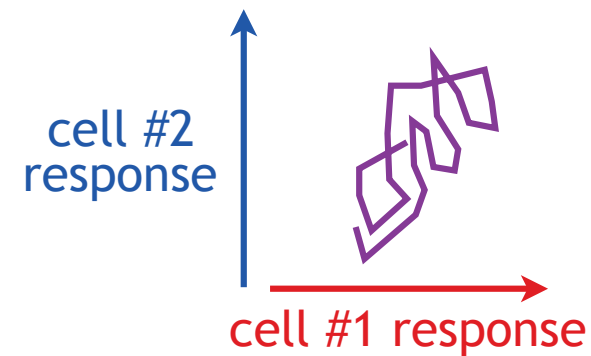


time



amortised inference:  
network parameters  
(synaptic weights, etc.)  
remain the same

$$P(\text{feature 1, feature 2} \mid \text{stimulus}) = P(\text{response 1, response 2} \mid \text{stimulus})$$



Fiser et al, TICS 2010

see also:

Hinton & Sejnowski, PDP 1986; Hinton et al, Science 1995; Dayan 1999;  
Hoyer & Hyvarinen, NIPS 2003, Lee & Mumford 2003

# NEURAL HALLMARKS OF SAMPLING



Gergő  
Orbán

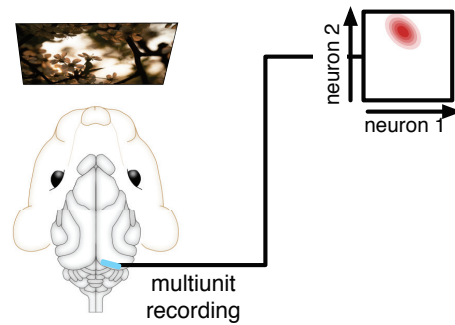


Pietro  
Berkes



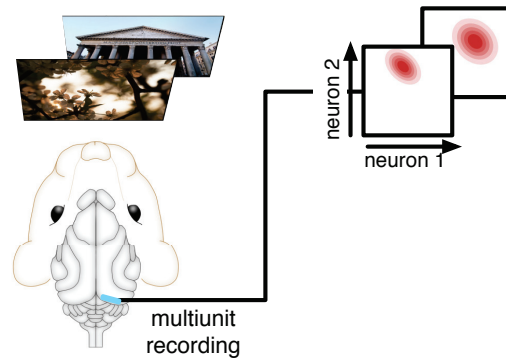
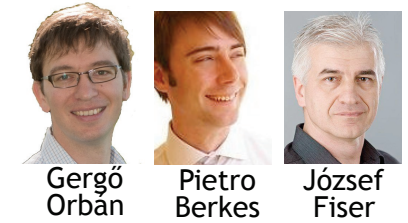
József  
Fiser

# NEURAL HALLMARKS OF SAMPLING



$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$

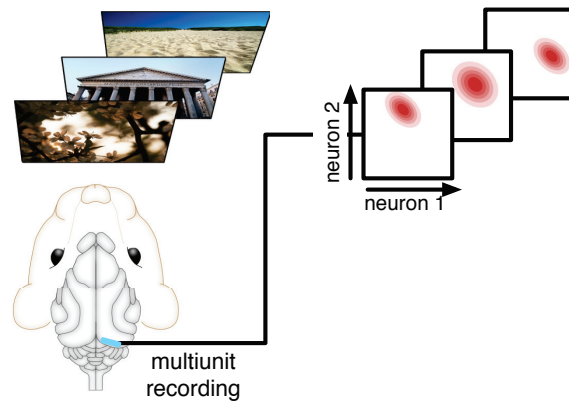
# NEURAL HALLMARKS OF SAMPLING



$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$
$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$

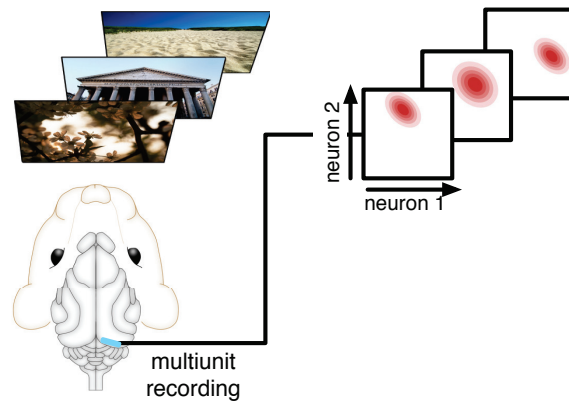
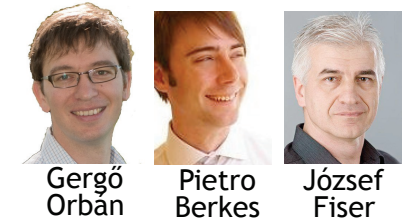


# NEURAL HALLMARKS OF SAMPLING



$$P_{\text{model}}(\text{feat}|\text{stim}_3)$$
$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$
$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$

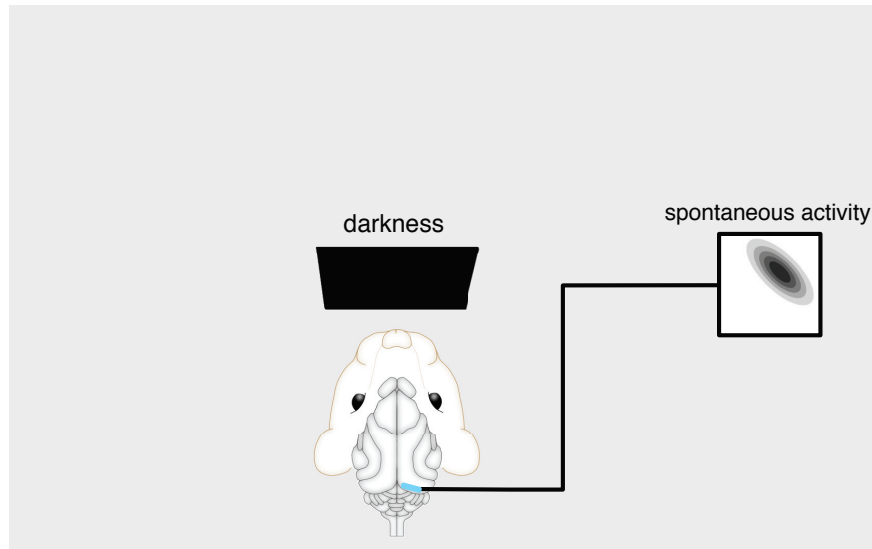
# NEURAL HALLMARKS OF SAMPLING



$$P_{\text{model}}(\text{feat}|\text{stim}_3)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$

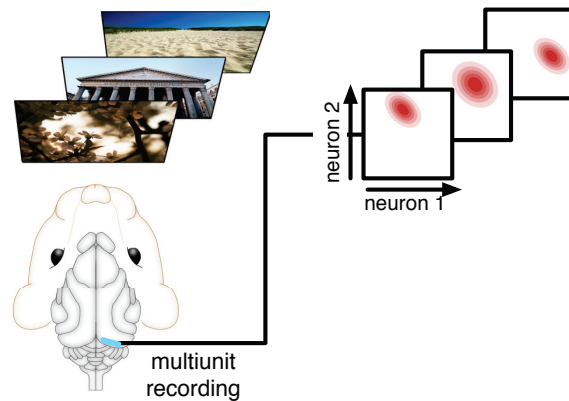
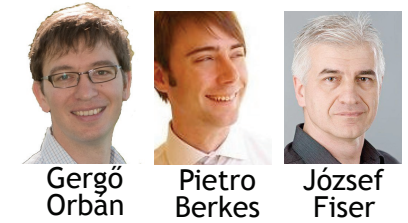
$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$



$$P_{\text{model}}(\text{feat}|\text{stim} = 0)$$

*Berkes et al, Science 2011*

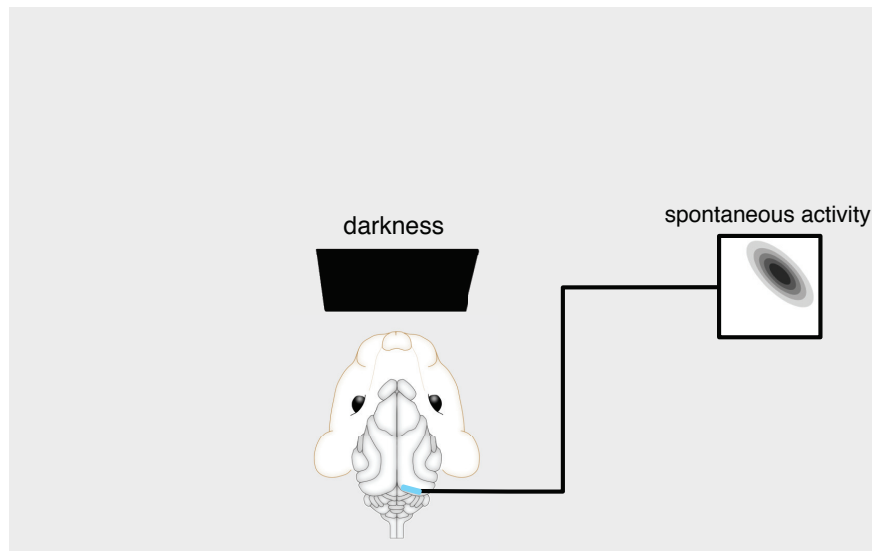
# NEURAL HALLMARKS OF SAMPLING



$$P_{\text{model}}(\text{feat}|\text{stim}_3)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$

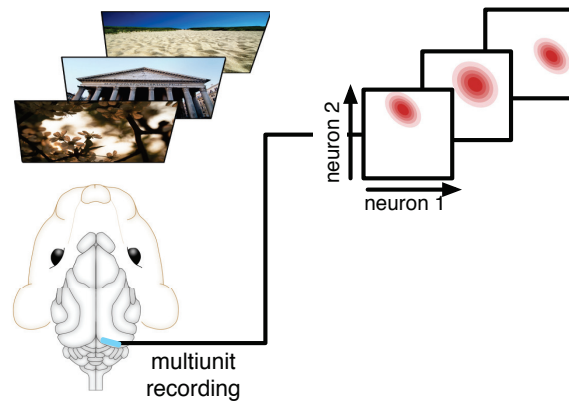
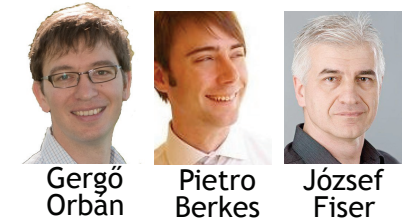
$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$



$$P_{\text{model}}(\text{feat}|\text{stim} = 0) \simeq P_{\text{model}}(\text{feat})$$

*Berkes et al, Science 2011*

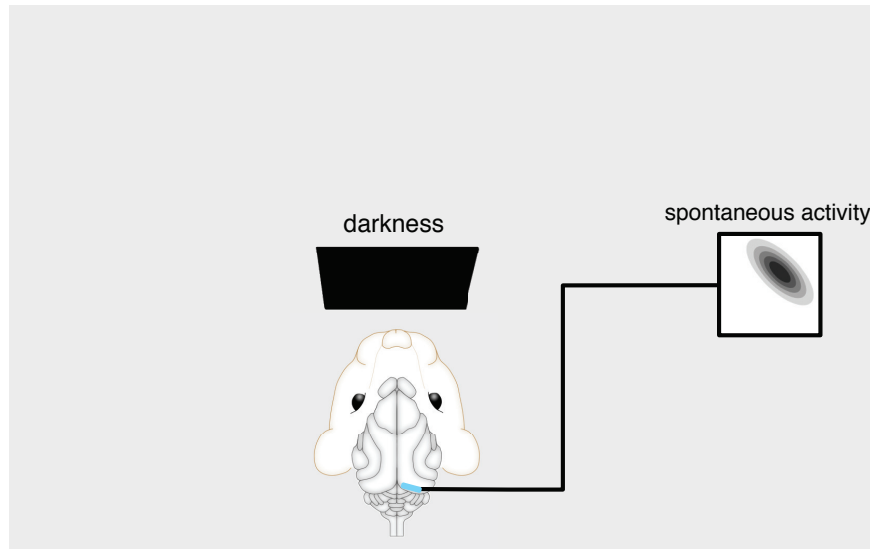
# NEURAL HALLMARKS OF SAMPLING



$$P_{\text{model}}(\text{feat}|\text{stim}_3)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$

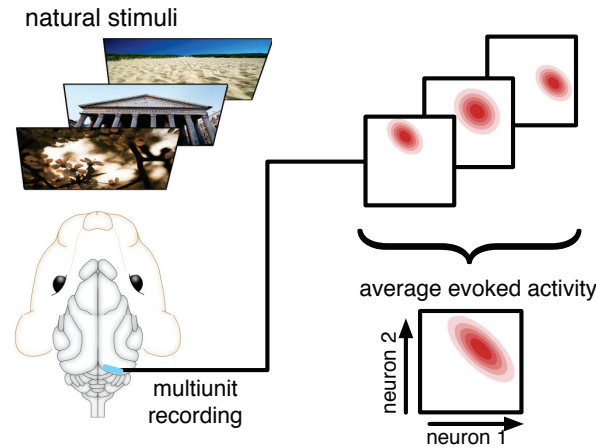


$$P_{\text{model}}(\text{feat}|\text{stim} = 0) \simeq P_{\text{model}}(\text{feat})$$

$$\simeq \int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{model}}(\text{stim}) d\text{stim}$$

*Berkes et al, Science 2011*

# NEURAL HALLMARKS OF SAMPLING

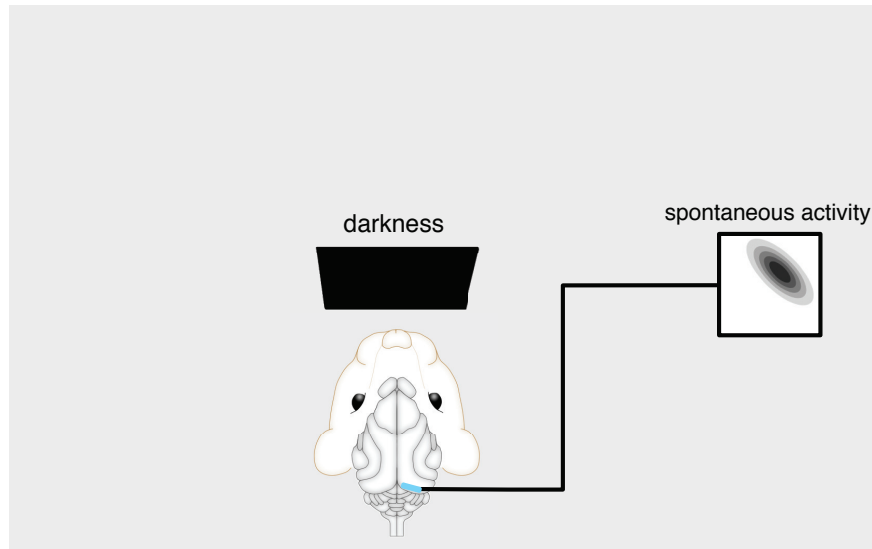


$$P_{\text{model}}(\text{feat}|\text{stim}_3)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$

$$\int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{natural}}(\text{stim}) d\text{stim}$$

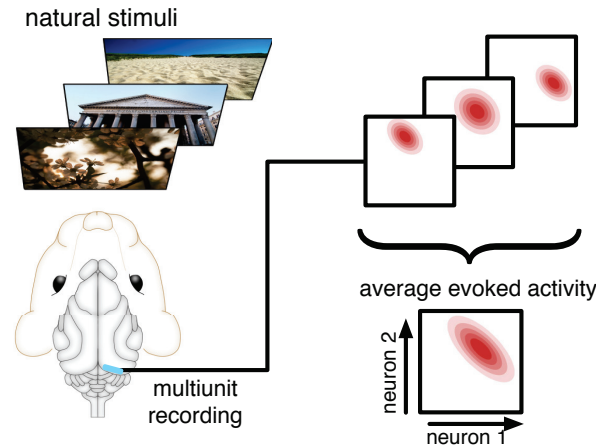


$$P_{\text{model}}(\text{feat}|\text{stim} = 0) \simeq P_{\text{model}}(\text{feat})$$

$$\simeq \int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{model}}(\text{stim}) d\text{stim}$$

*Berkes et al, Science 2011*

# NEURAL HALLMARKS OF SAMPLING

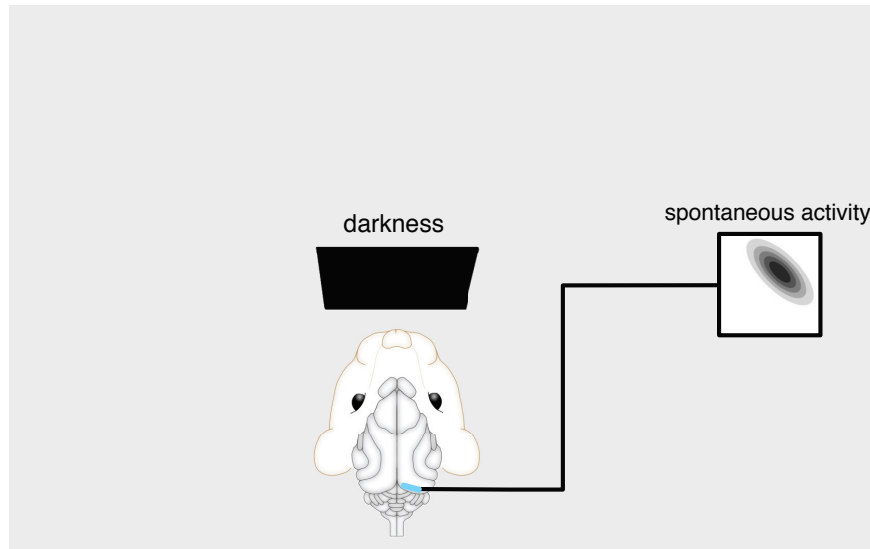


$$P_{\text{model}}(\text{feat}|\text{stim}_3)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$

$$\int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{natural}}(\text{stim}) d\text{stim}$$



$$P_{\text{model}}(\text{feat}|\text{stim} = 0) \simeq P_{\text{model}}(\text{feat})$$

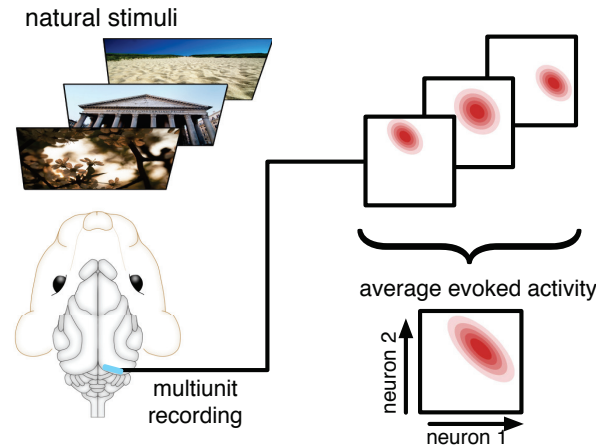
$$\int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{model}}(\text{stim}) d\text{stim}$$

Berkes et al, Science 2011

# NEURAL HALLMARKS OF SAMPLING

+ STATISTICALLY OPTIMAL ADAPTATION

$$P_{\text{model}}(\text{stim}) = P_{\text{natural}}(\text{stim})$$

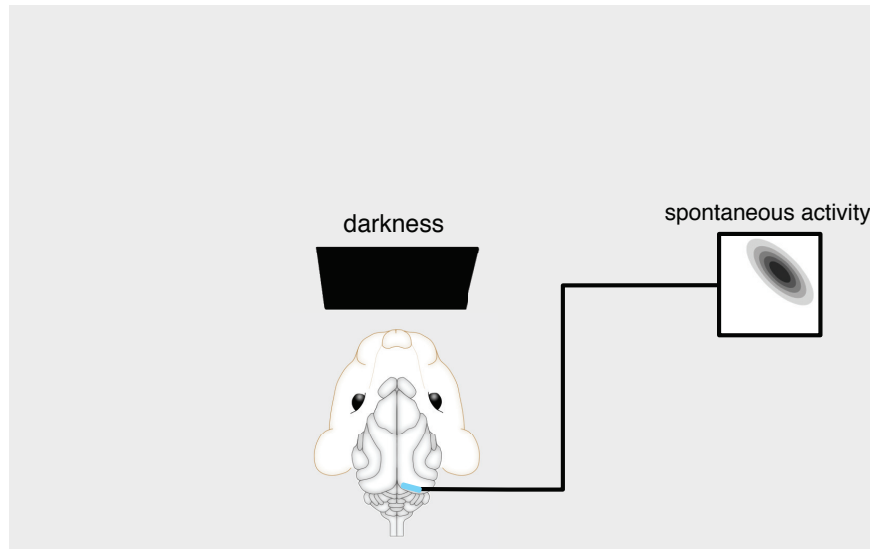


$$P_{\text{model}}(\text{feat}|\text{stim}_3)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$

$$\int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{natural}}(\text{stim}) d\text{stim}$$



$$P_{\text{model}}(\text{feat}|\text{stim} = 0) \simeq P_{\text{model}}(\text{feat})$$

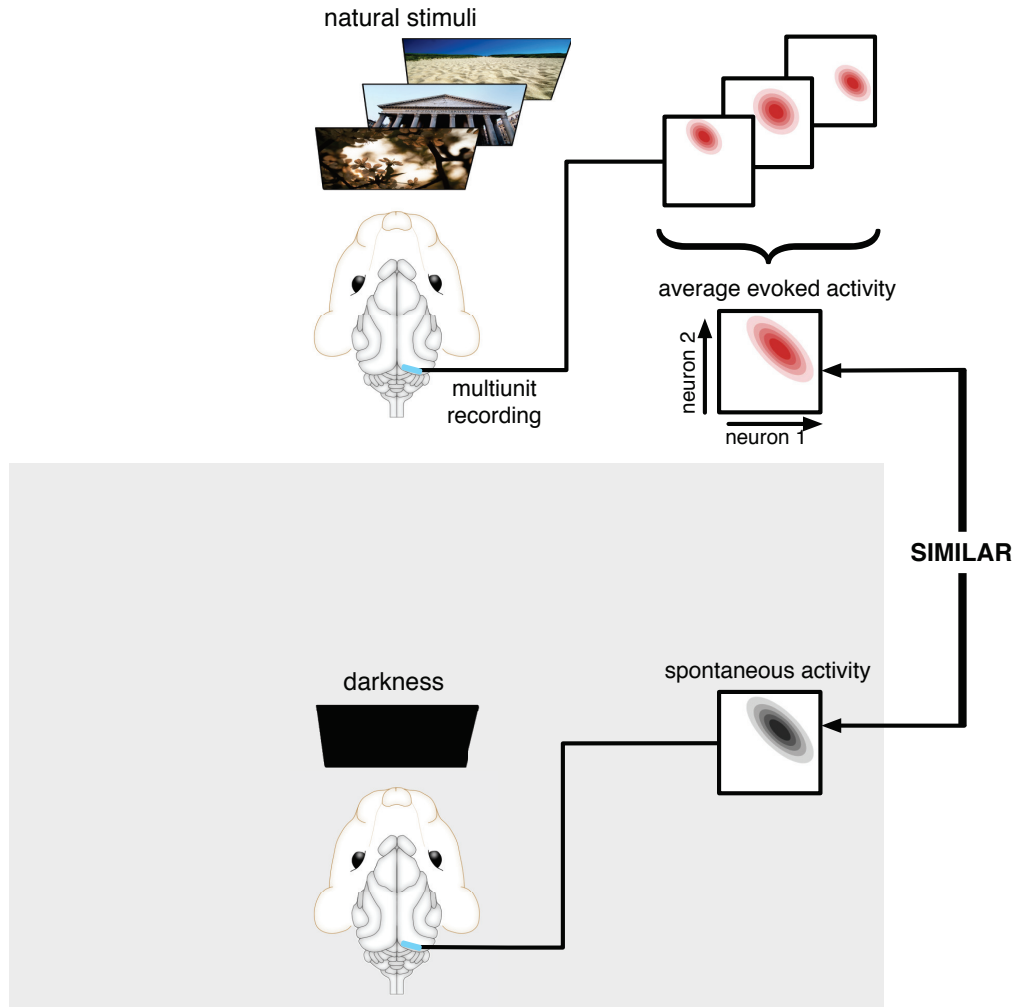
$$\int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{model}}(\text{stim}) d\text{stim}$$

Berkes et al, Science 2011

# NEURAL HALLMARKS OF SAMPLING

+ STATISTICALLY OPTIMAL ADAPTATION

$$P_{\text{model}}(\text{stim}) = P_{\text{natural}}(\text{stim})$$



$$P_{\text{model}}(\text{feat}|\text{stim}_3)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$

$$\int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{natural}}(\text{stim}) d\text{stim}$$

$$P_{\text{model}}(\text{feat}|\text{stim} = 0) \simeq P_{\text{model}}(\text{feat})$$

$$\approx \int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{model}}(\text{stim}) d\text{stim}$$

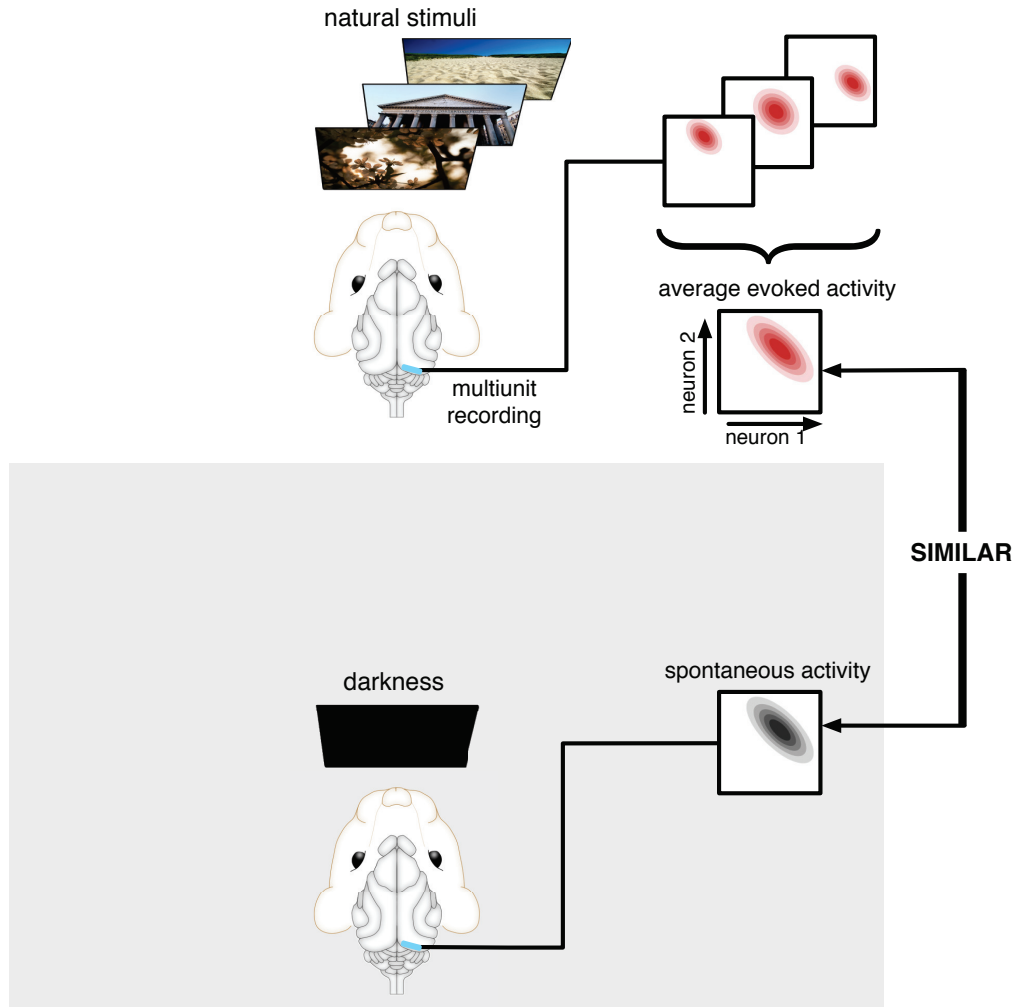
Berkes et al, Science 2011



# NEURAL HALLMARKS OF SAMPLING

+ STATISTICALLY OPTIMAL ADAPTATION

$$P_{\text{model}}(\text{stim}) = P_{\text{natural}}(\text{stim})$$



$$P_{\text{model}}(\text{feat}|\text{stim}_3)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$

$$\int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{natural}}(\text{stim}) d\text{stim}$$

**average inference**  
**=**  
**prior expectations**

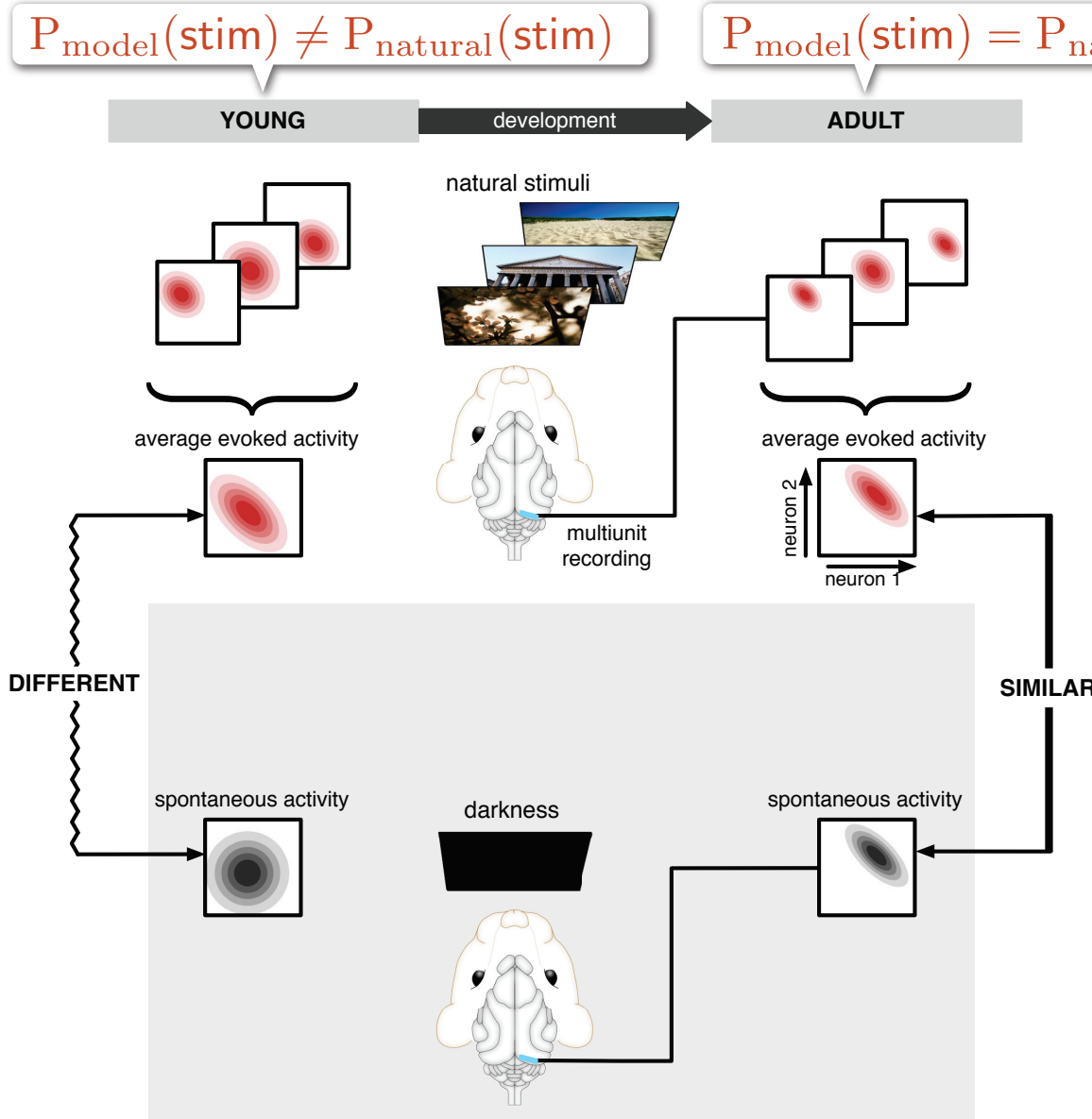
$$P_{\text{model}}(\text{feat}|\text{stim} = 0) \simeq P_{\text{model}}(\text{feat})$$

$$\approx \int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{model}}(\text{stim}) d\text{stim}$$

Berkes et al, Science 2011

# NEURAL HALLMARKS OF SAMPLING

+ STATISTICALLY OPTIMAL ADAPTATION



$$P_{\text{model}}(\text{feat}|\text{stim}_3)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_2)$$

$$P_{\text{model}}(\text{feat}|\text{stim}_1)$$

$$\int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{natural}}(\text{stim}) d\text{stim}$$

**average inference**  
=  
**prior expectations**

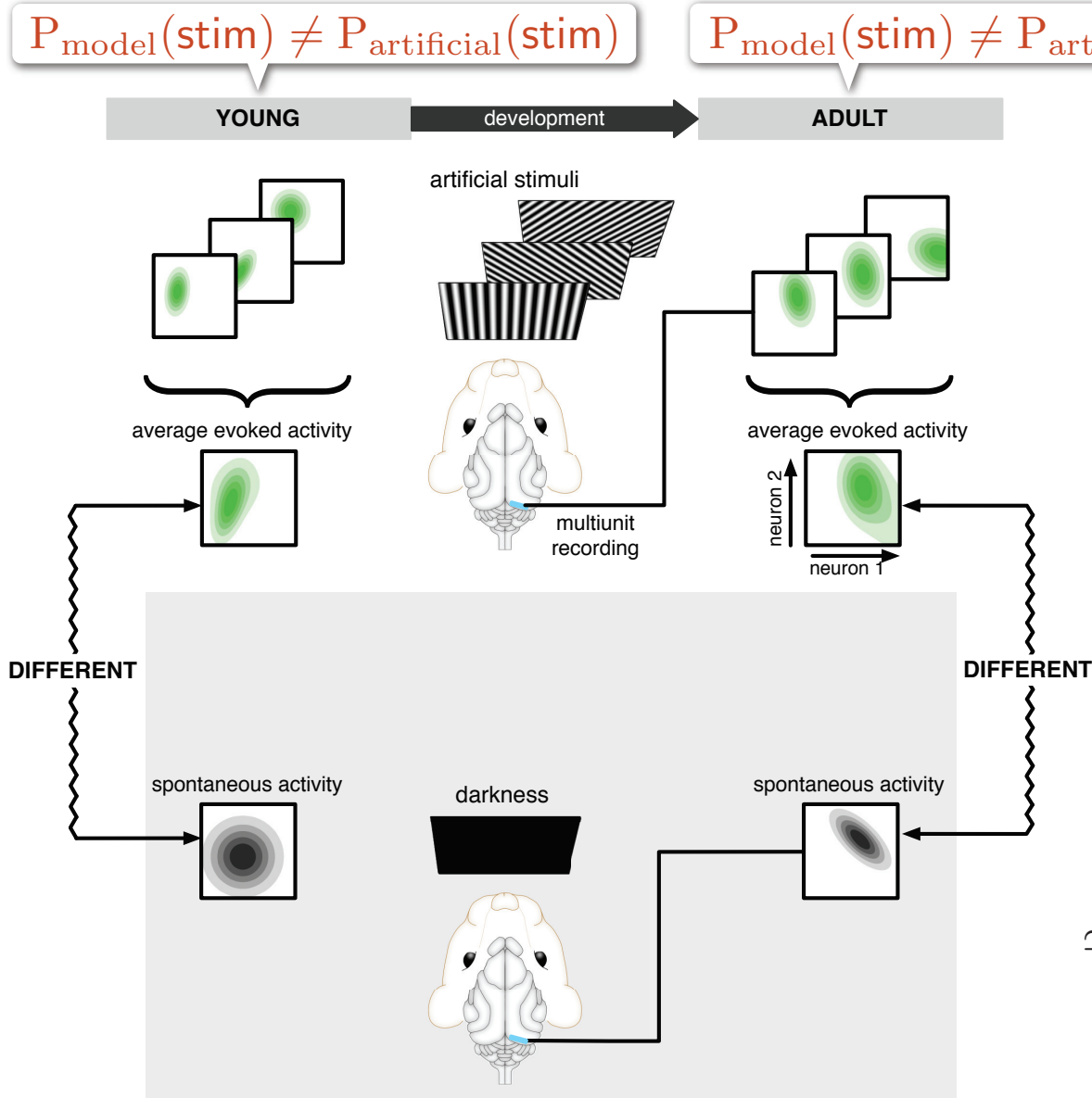
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Berkes et al, Science 2011

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Berkes et al, Science 2011

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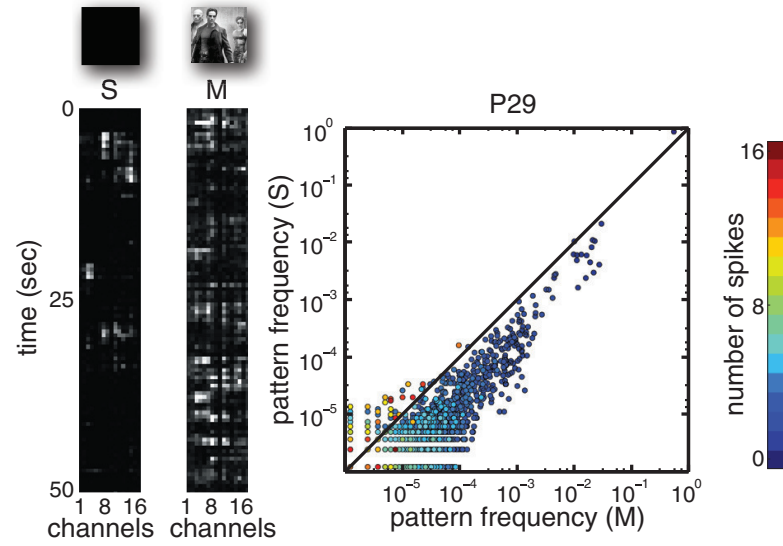
$$\int P_{\text{model}}(\text{feat}|\text{stim}) P_{\text{artificial}}(\text{stim}) d\text{stim}$$

average inference  
 $\neq$   
prior expectations

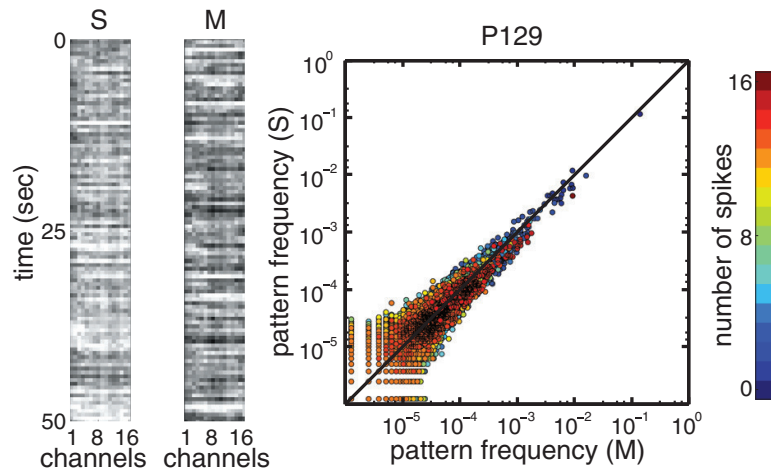
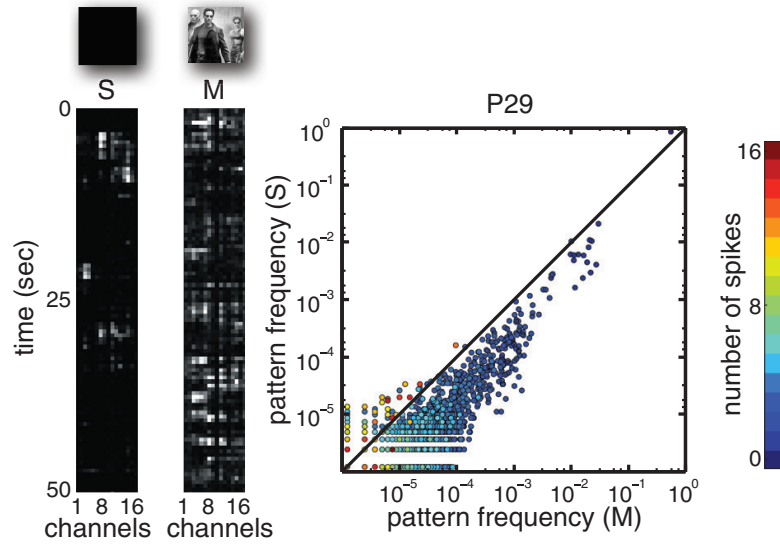
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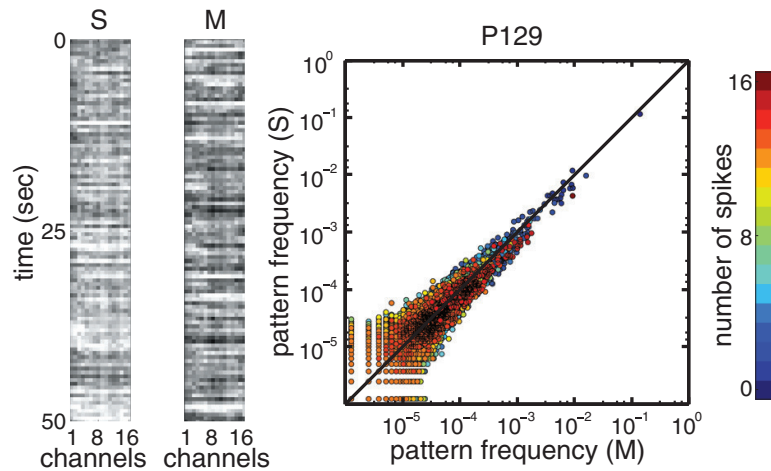
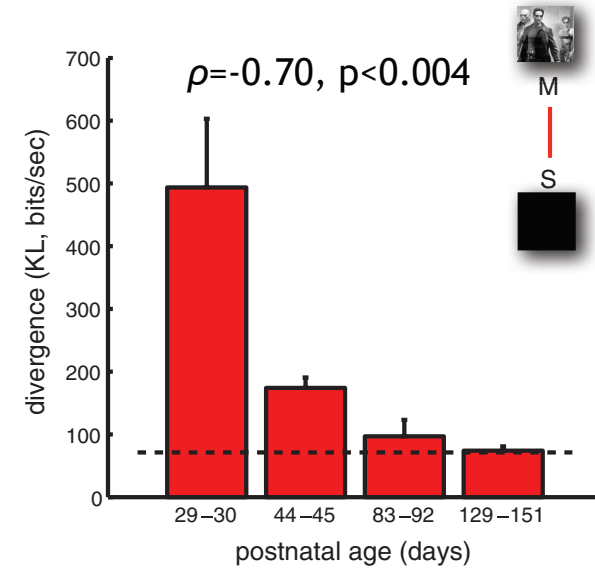
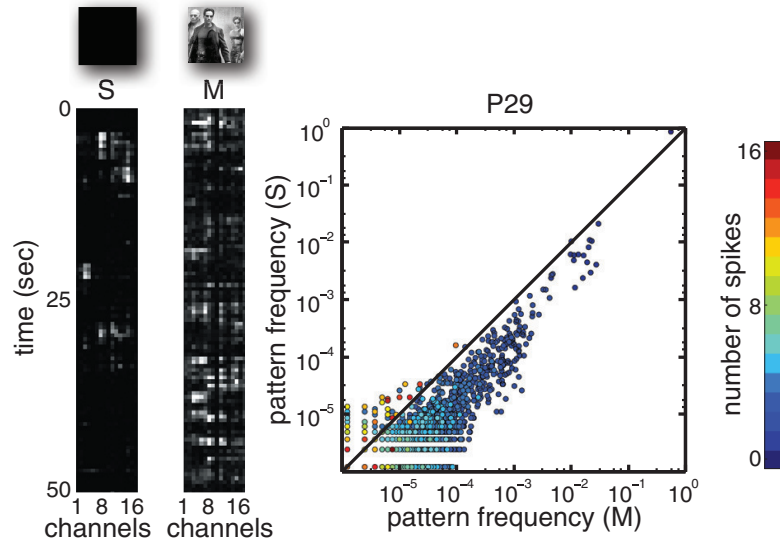
# DEVELOPMENTAL CHANGES



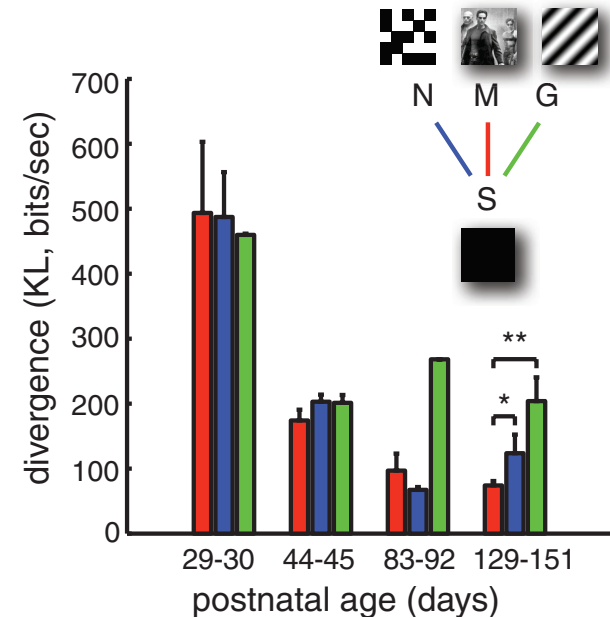
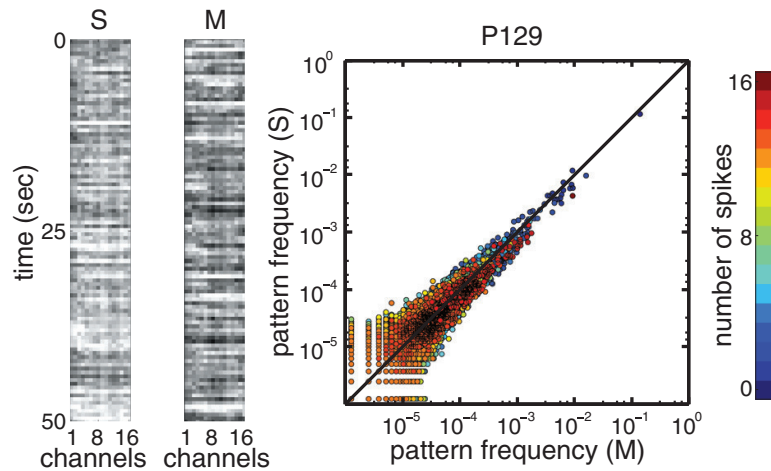
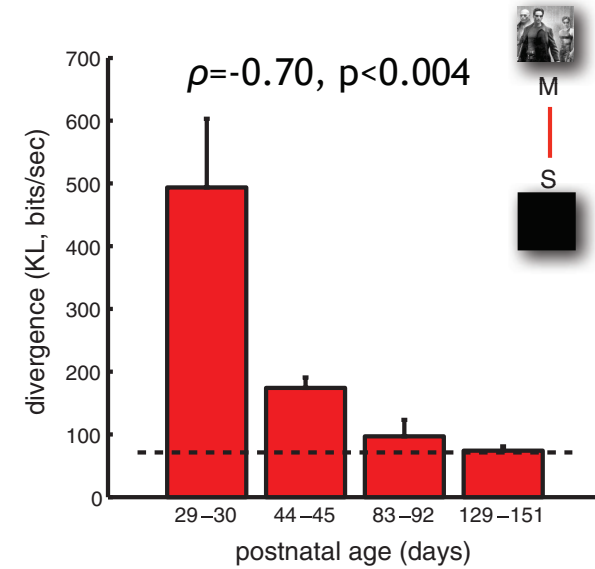
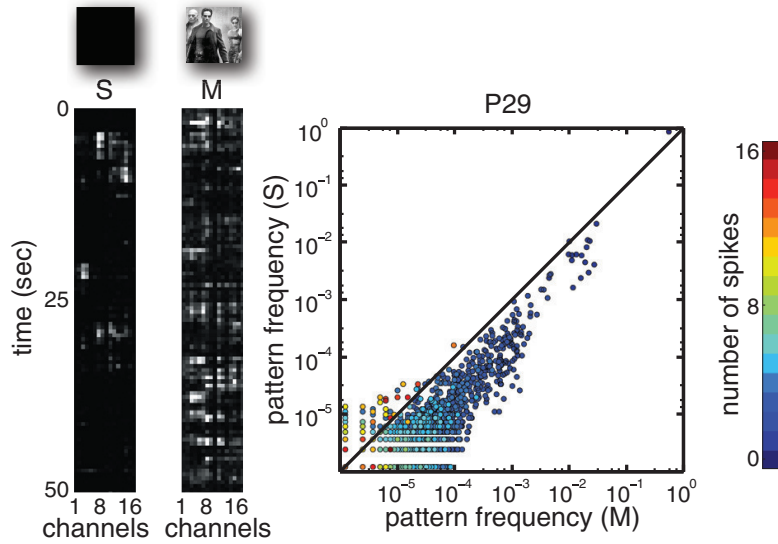
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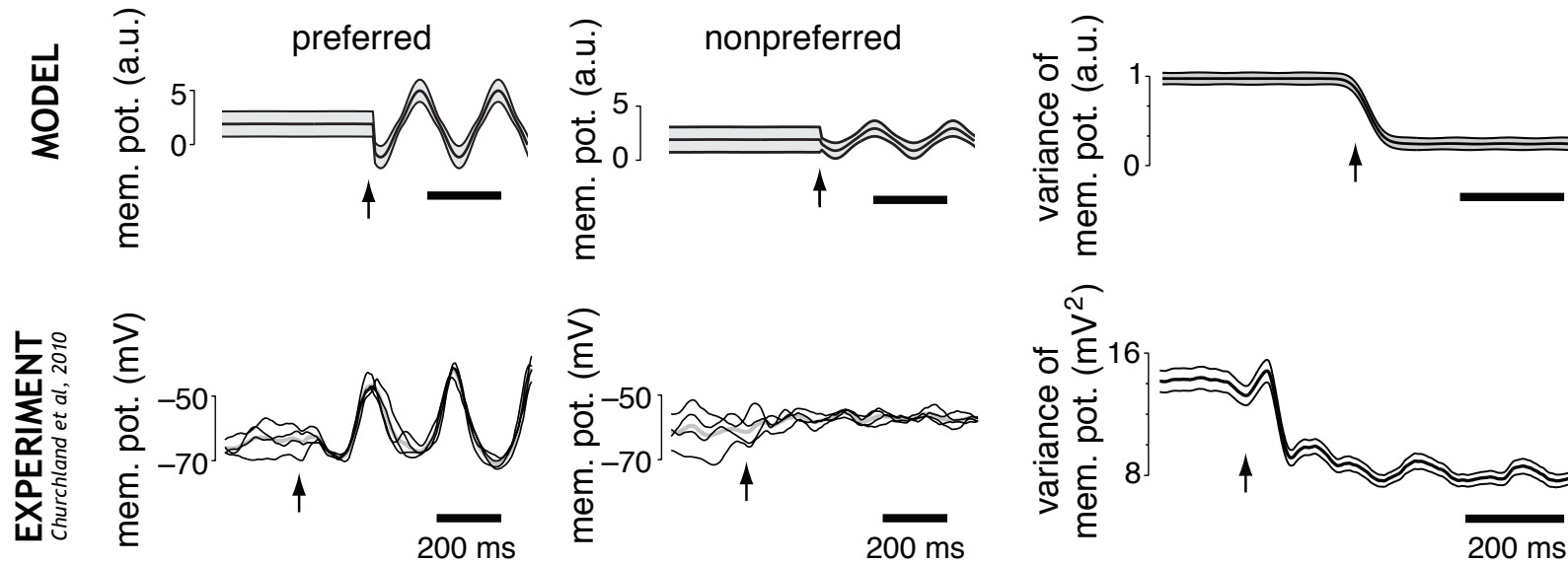


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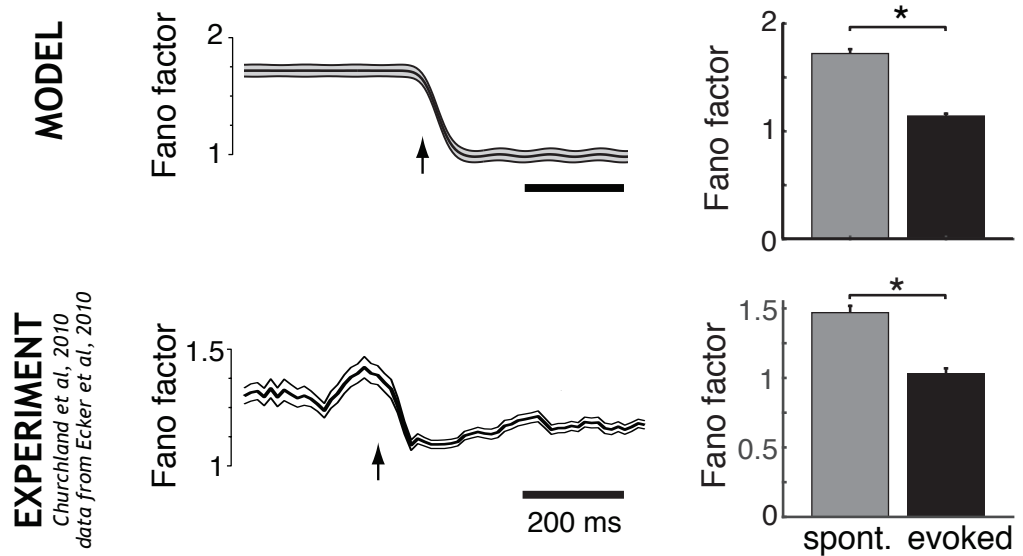
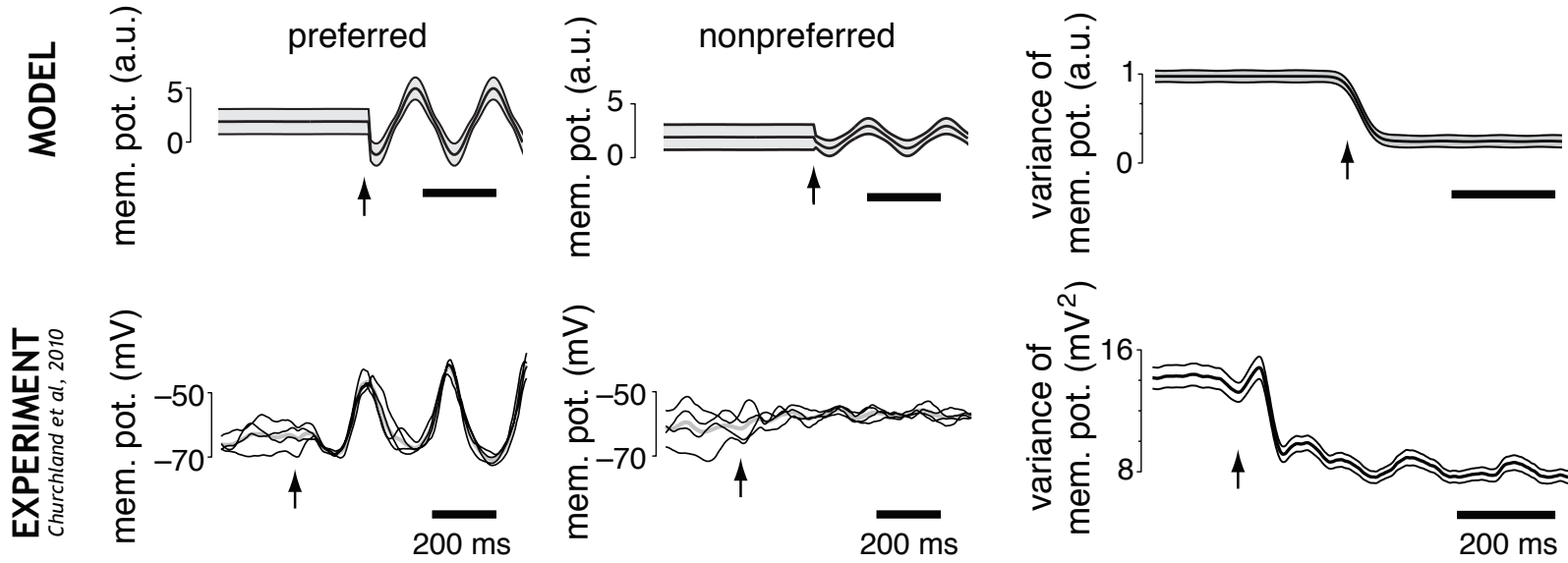
*Berkes et al, Science 2011*

# STIMULUS ONSET $\Rightarrow$ CONTRAST $\uparrow$ $\Rightarrow$ NOISE VARIABILITY $\downarrow$



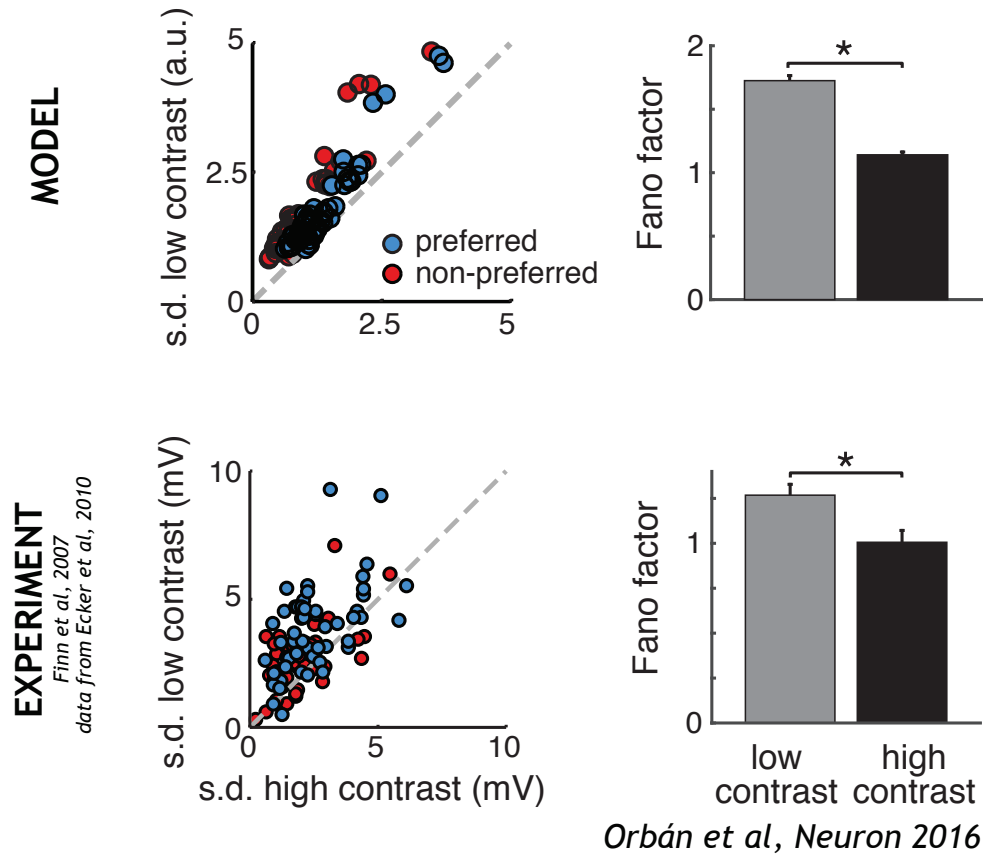


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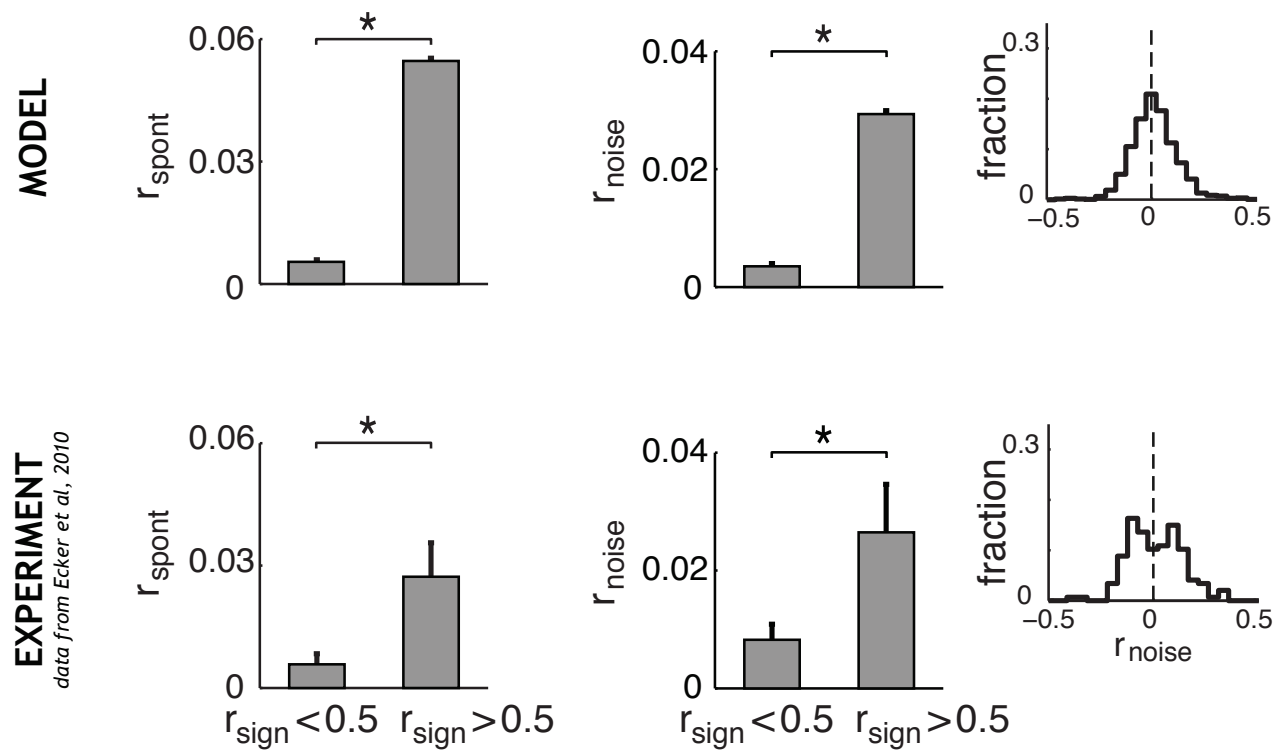


*Orbán et al, Neuron 2016*

# CONTRAST $\uparrow$ $\Rightarrow$ NOISE VARIABILITY $\downarrow$



# SPONTANEOUS $\approx$ SIGNAL $\approx$ NOISE CORRELATIONS



Orbán et al, Neuron 2016

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low-level visual features, abstract shapes, faces

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evoked and spontaneous neural activity  
(so far: stationary statistics → circuit dynamics?)



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# ACKNOWLEDGEMENTS



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Fiser



Richard  
Aslin

data sharing:

Alexander Ecker  
Matthias Bethge  
Andreas Tolias

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Andreas Tolias

wellcome trust

Investigator





# POSTDOC POSITION AVAILABLE

estimate humans' complex, high dimensional,  
dynamically changing internal models from behaviour

collaborate with world-leading  
experimental cognitive scientists

