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Donders Institute
for Brain, Cognition and Behaviour

# Why cognitive scientists should care about computational complexity

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Computational
Cognitive Science

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# Inspiration

## Why Philosophers Should Care About Computational Complexity

*Scott Aaronson*

One might think that, once we know something is computable, how efficiently it can be computed is a practical question with little further philosophical importance. In this essay, I offer a detailed case that one would be wrong. In particular, I argue that computational complexity theory—the field that studies the resources (such as time, space, and randomness) needed to solve computational problems—leads to new perspectives (…) the strong AI debate, computationalism, the problem of logical omniscience, Hume's problem of induction, Goodman's grue riddle, the foundations of quantum mechanics, economic rationality (…).

# Inspiration

## Why **Philosophers** Should Care About Computational Complexity

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One might think that, once we know something is computable, how efficiently it can be computed is a practical question with little further philosophical importance. In this essay, I offer a detailed case that one would be wrong. In particular, I argue that computational complexity theory—the field that studies the resources (such as time, space, and randomness) needed to solve computational problems—leads to new perspectives on (…) **the strong AI debate, computationalism, the problem of logical omniscience, Hume's problem of induction, Goodman's grue riddle, the foundations of quantum mechanics, economic rationality** (…)

# Why Cognitive Scientists Should Care About Computational Complexity

*Iris van Rooij*

You may think you need not care much.

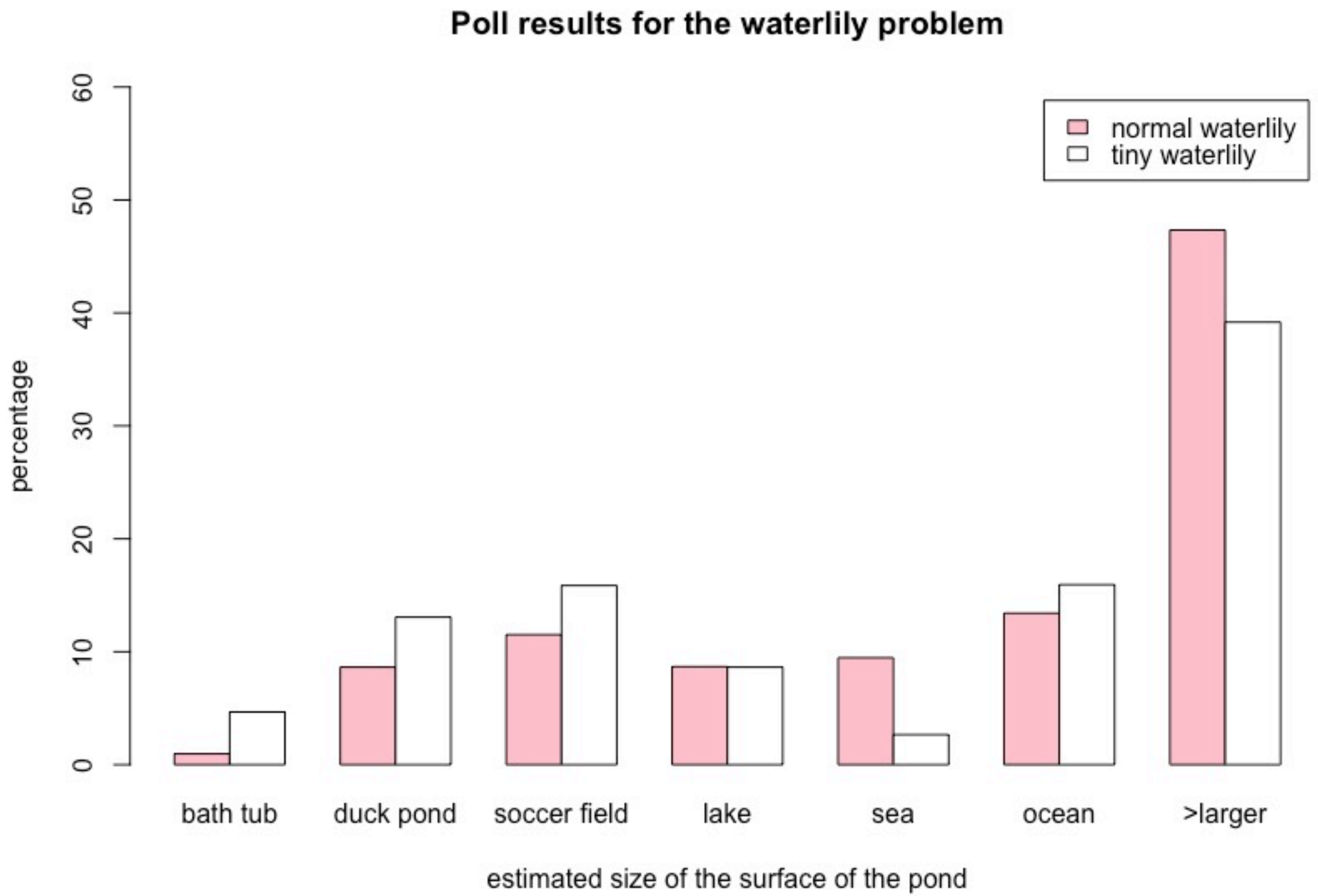
But you'd be wrong.

# Water lilies

**Pick the option closest to your intuition** (no calculations please).

(1) Smaller than any below
(2) Bath tub
(3) Duck pond
(4) Soccer field
(5) Lake
(6) Sea
(7) Ocean
(8) Larger than any above

# Water lilies



# Intuitions can be mistaken

# Intuitions can be mistaken

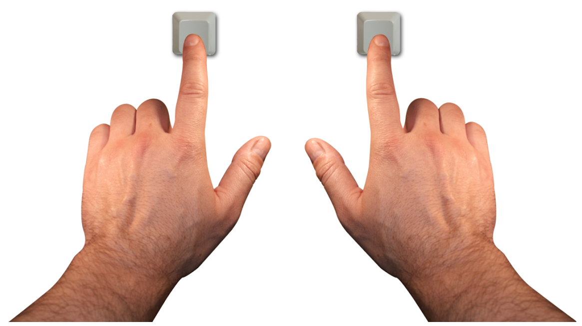
- **Whether or not models scale to the real world?**
- One may incorrectly intuit that if a model works for a toy domain then it will work (approximately) in the real world.

- **What will make models scale better?**
- One may incorrectly intuit that satisficing, heuristics, modularity, extended cognition, etc. etc. can make intractable computations scale (approximately).

# Scaling models of cognition

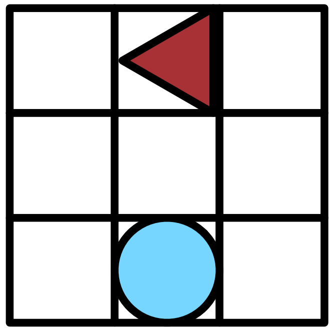
Model *M* → scaling → Model *M*

## Decision making



← experimentation

## Communication



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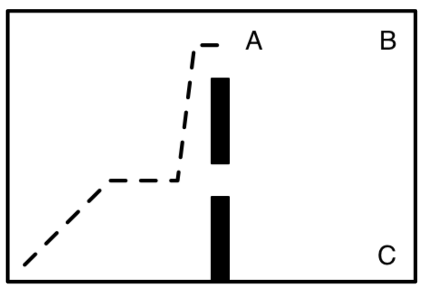
# Scaling models of cognition

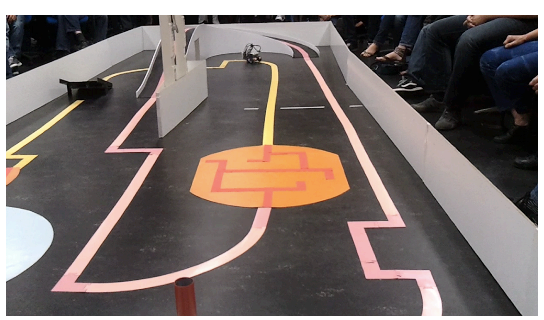
Model *M* → scaling → Model *M*

## Action understanding




← experimentation

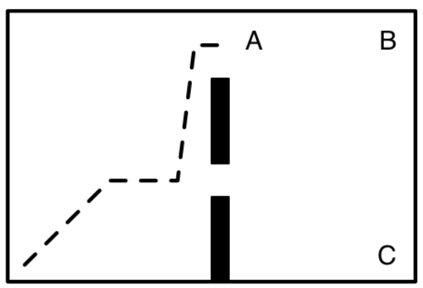
## Navigation (Robotics)



← experimentation

# Scaling models of cognition


## Action understanding




experimentation

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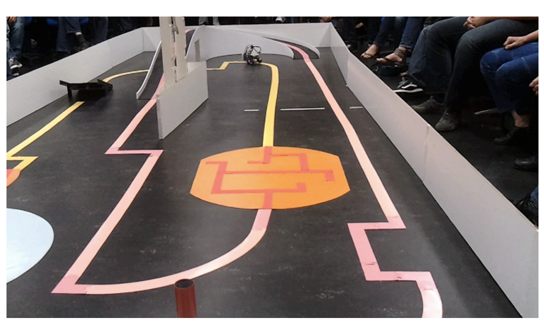
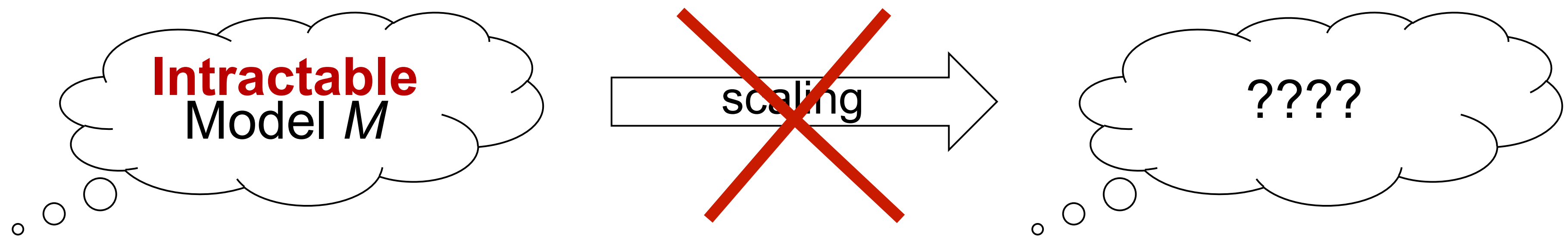
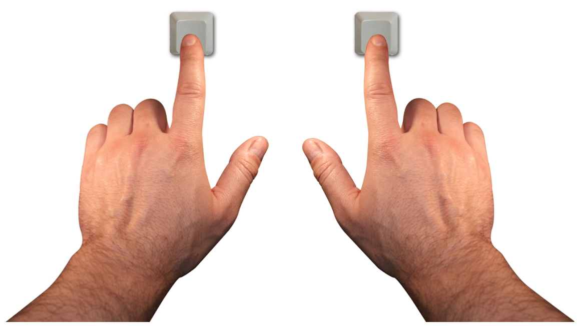


experimentation

# Scaling models of cognition

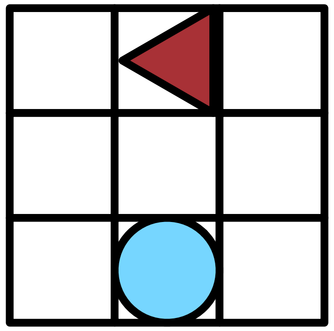



Decision making



experimentation

Communication



experimentation

## What is Intractability? And why is it a problem?

"The computations postulated by a model of cognition need to be tractable in the real world in which people live, not only in the small world of an experiment ... This eliminates NP-hard models that lead to computational explosion." (Gigerenzer et al., 2008)

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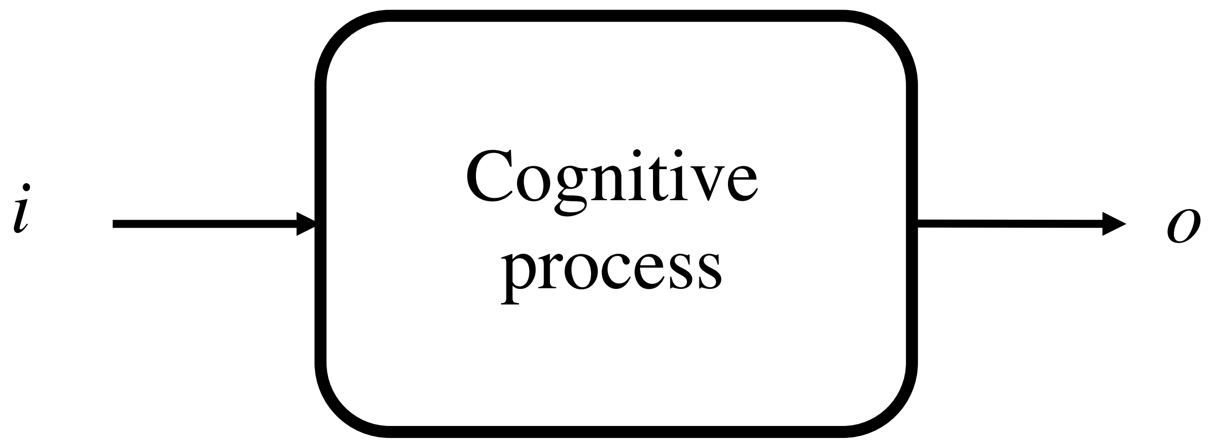
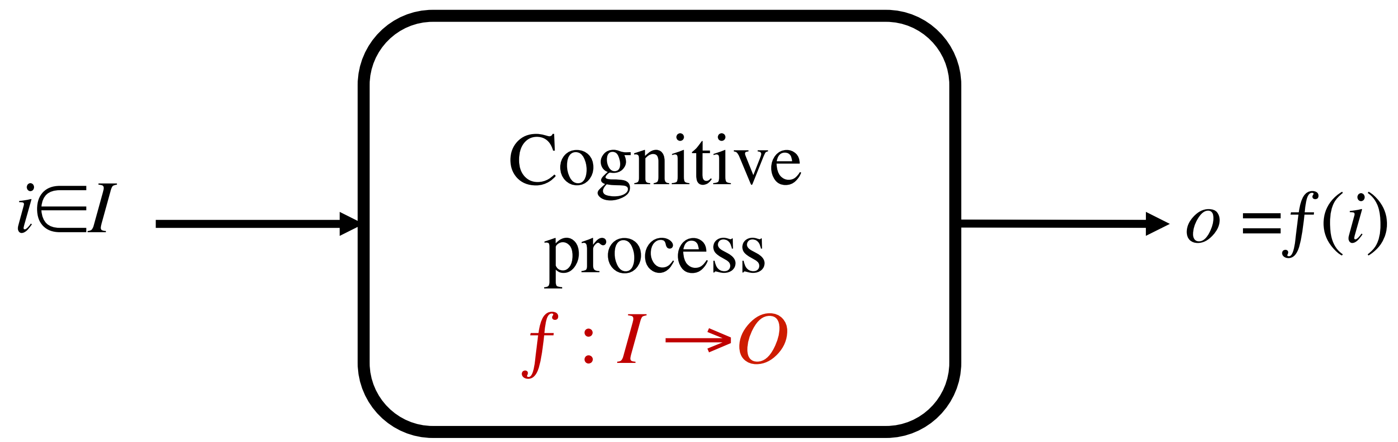
# Cognitive explanation: 3 Levels of Marr




| Level | Marr's levels | Question |
|---|---|---|
| 1 | Computational | What? |
| 2 | Algorithm | How? |
| 3 | Implementation | Realisation? |

# Cognitive explanation: 3 Levels of Marr

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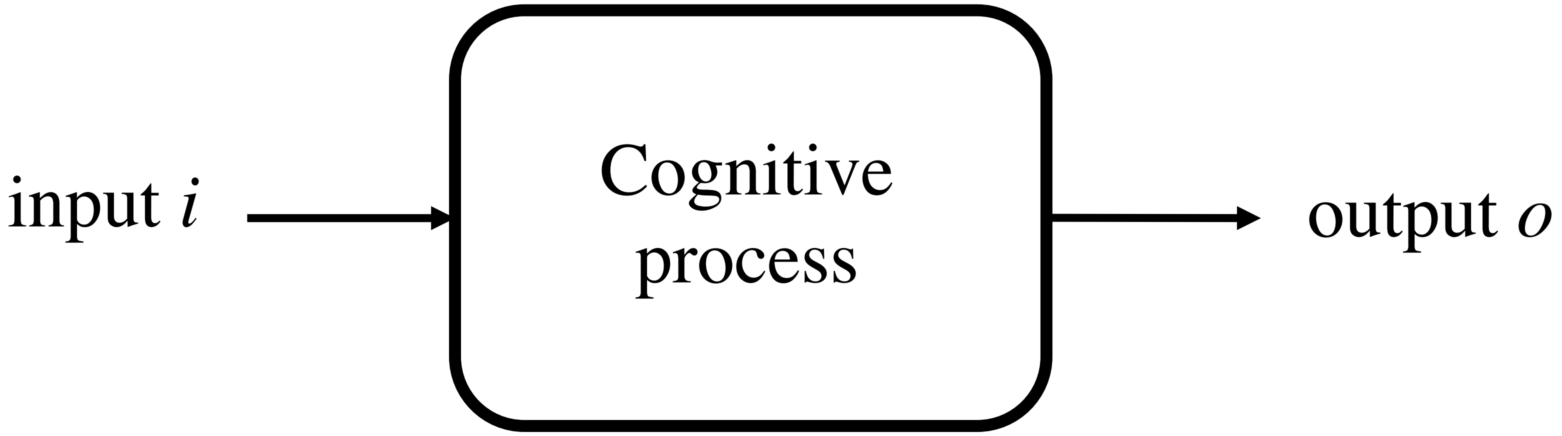
# Computational-level Models of Cognition



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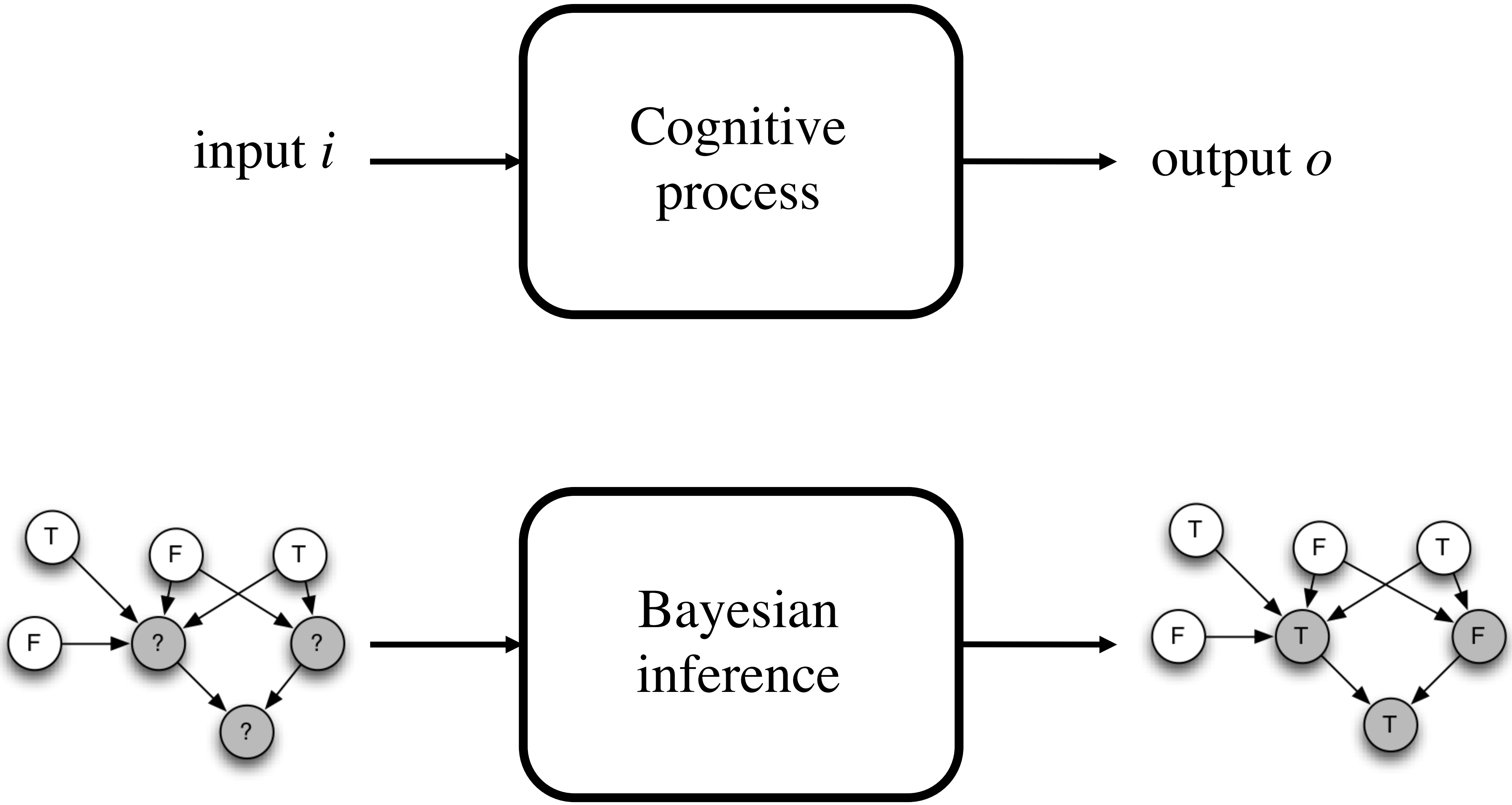

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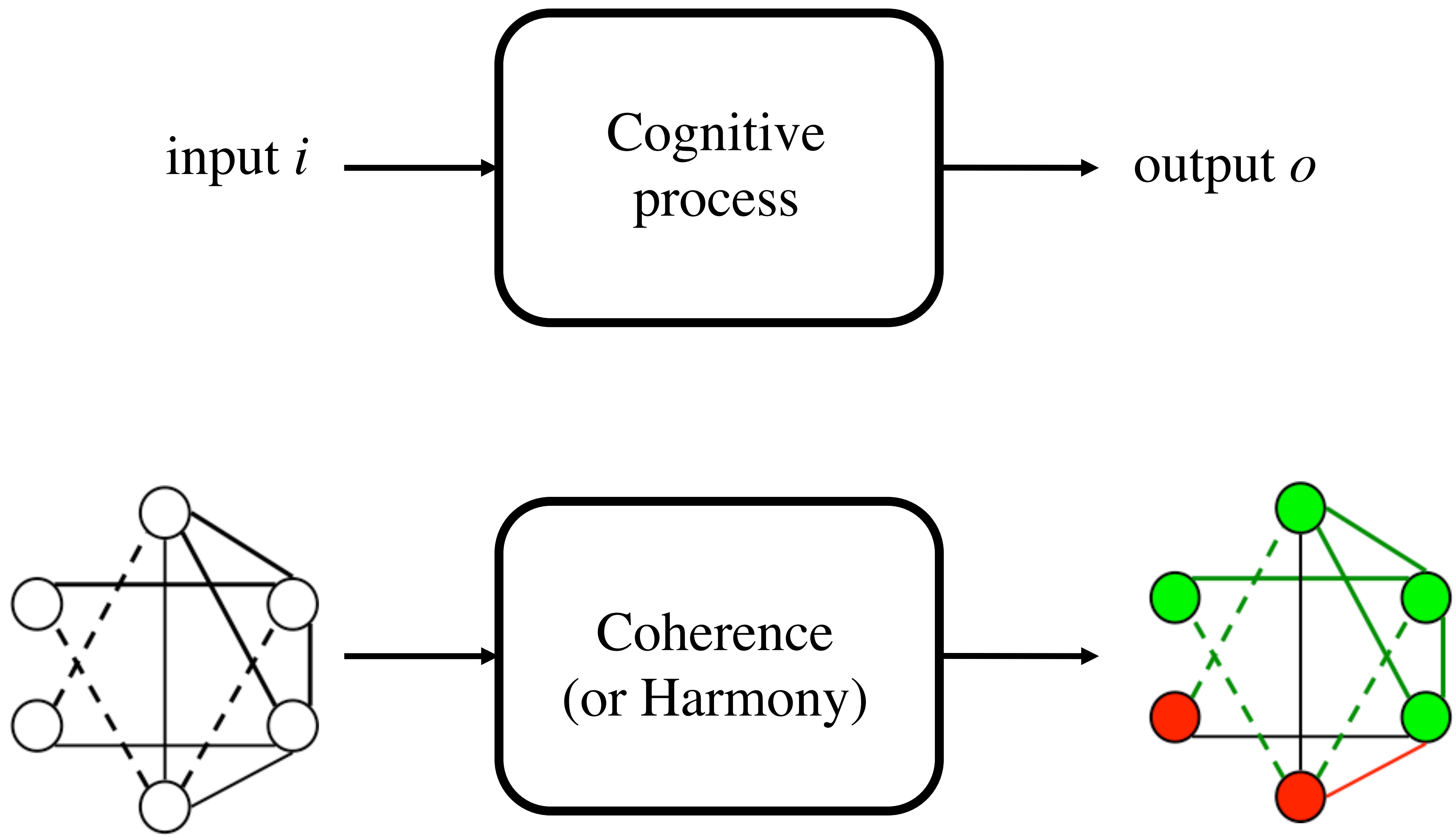

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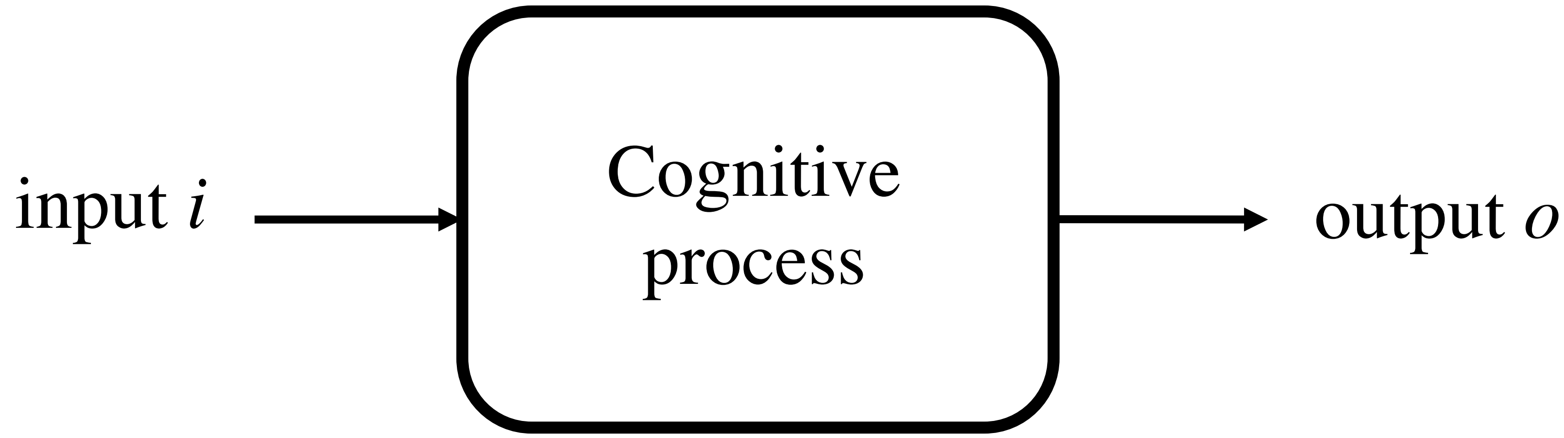


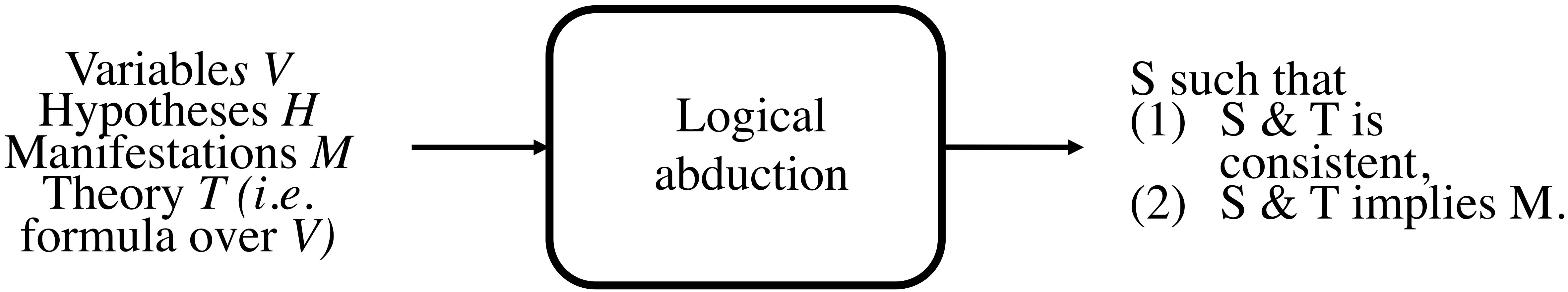

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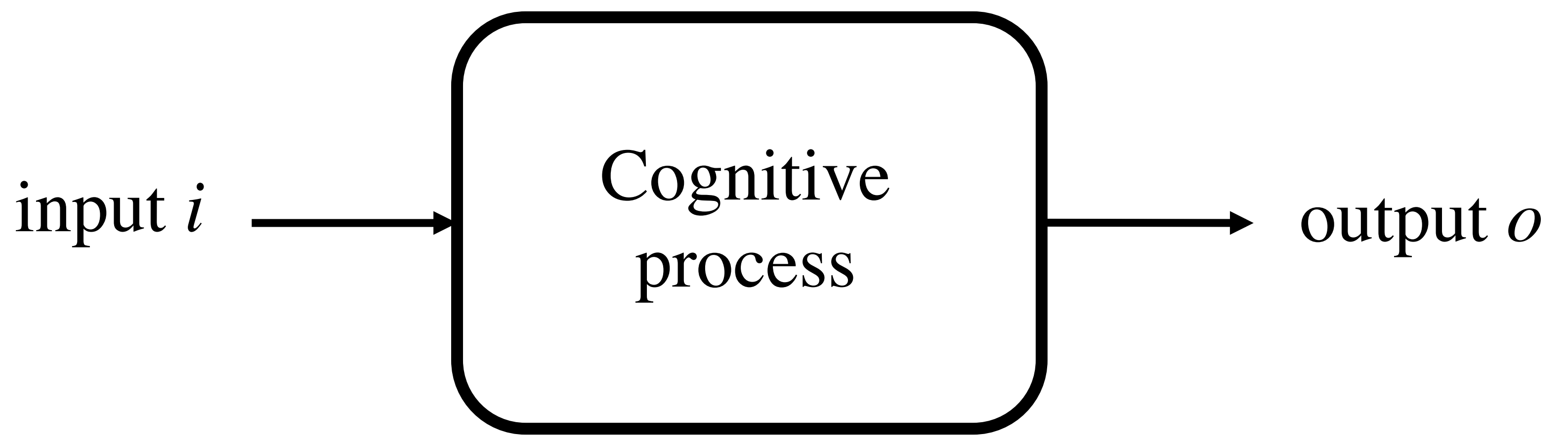
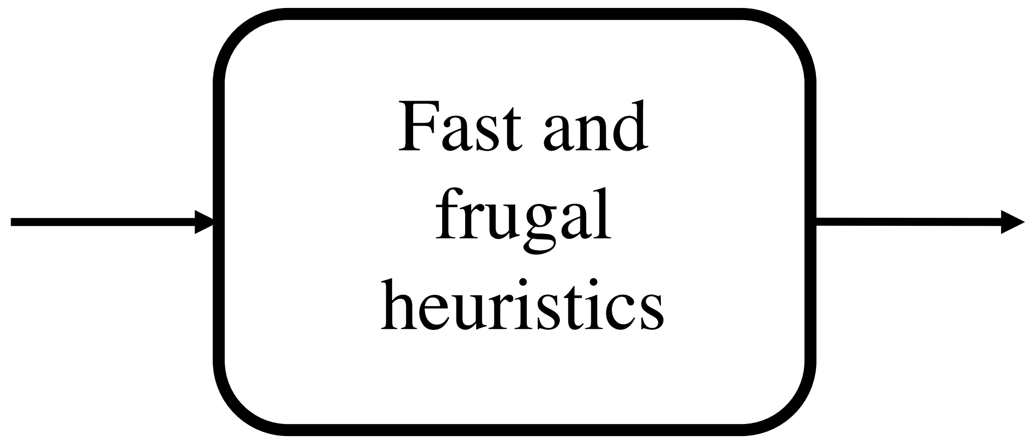
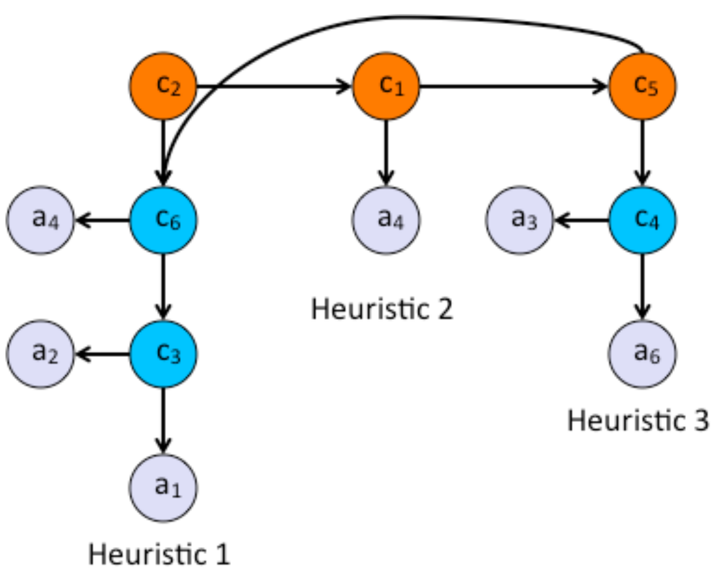



| | events | | | | | actions | | | |
|---|---|---|---|---|---|---|---|---|---|
| situation | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
| $s_1$ | T | F | F | F | T | 1 | 0 | 0 | 1 |
| $s_2$ | F | T | T | F | F | 1 | 0 | 1 | 1 |
| $s_2$ | F | F | F | T | F | 0 | 0 | 0 | 1 |
| $s_3$ | T | T | T | T | F | 1 | 0 | 0 | 1 |
| $s_4$ | F | T | F | T | F | 1 | 1 | 1 | 0 |
| $s_5$ | T | T | F | F | F | 1 | 1 | 1 | 1 |
| $s_6$ | F | F | T | F | F | 1 | 0 | 0 | 0 |



# What is Intractability? And why is it a problem?

"The computations postulated by a model of cognition need to be tractable in the real world in which people live, not only in the small world of an experiment ... This eliminates NP-hard models that lead to computational explosion." (Gigerenzer et al., 2008)

# Why NP-hard is considered intractable

NP-hard functions cannot be computed in polynomial time (assuming P $\neq$ NP). Instead they require exponential time (or worse) for their computation, which is why they are considered intractable (in other words, unrealistic to compute for all but small inputs).

| $n$ | $n^2$ |
|---|---|
| 5 | 0.15 msec |
| 20 | 0.04 sec |
| 50 | 0.25 sec |
| 100 | 1.00 sec |
| 1000 | 1.67 min |

| $n$ | $2^n$ |
|---|---|
| 5 | 0.19 msec |
| 20 | 1.75 min |
| 50 | $8.4 \times 10^2$ yrs |
| 100 | $9.4 \times 10^{17}$ yrs |
| 1000 | $7.9 \times 10^{288}$ yrs |

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polynomial

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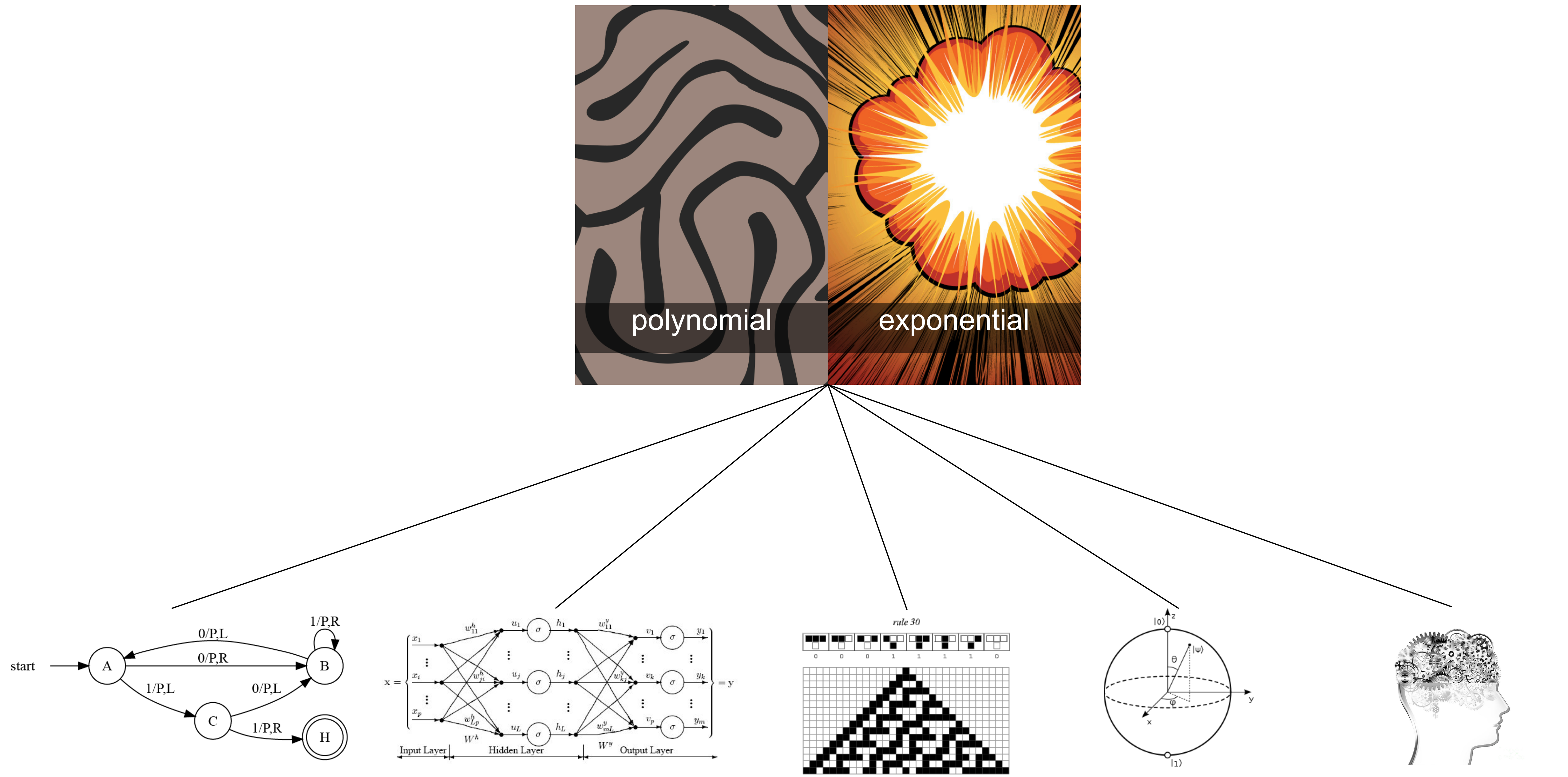
exponential

# Invariance Thesis




Turing Machine | Neural Network | Cellular Automata | Quantum Computer | Computational Brain Models

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# The Problem

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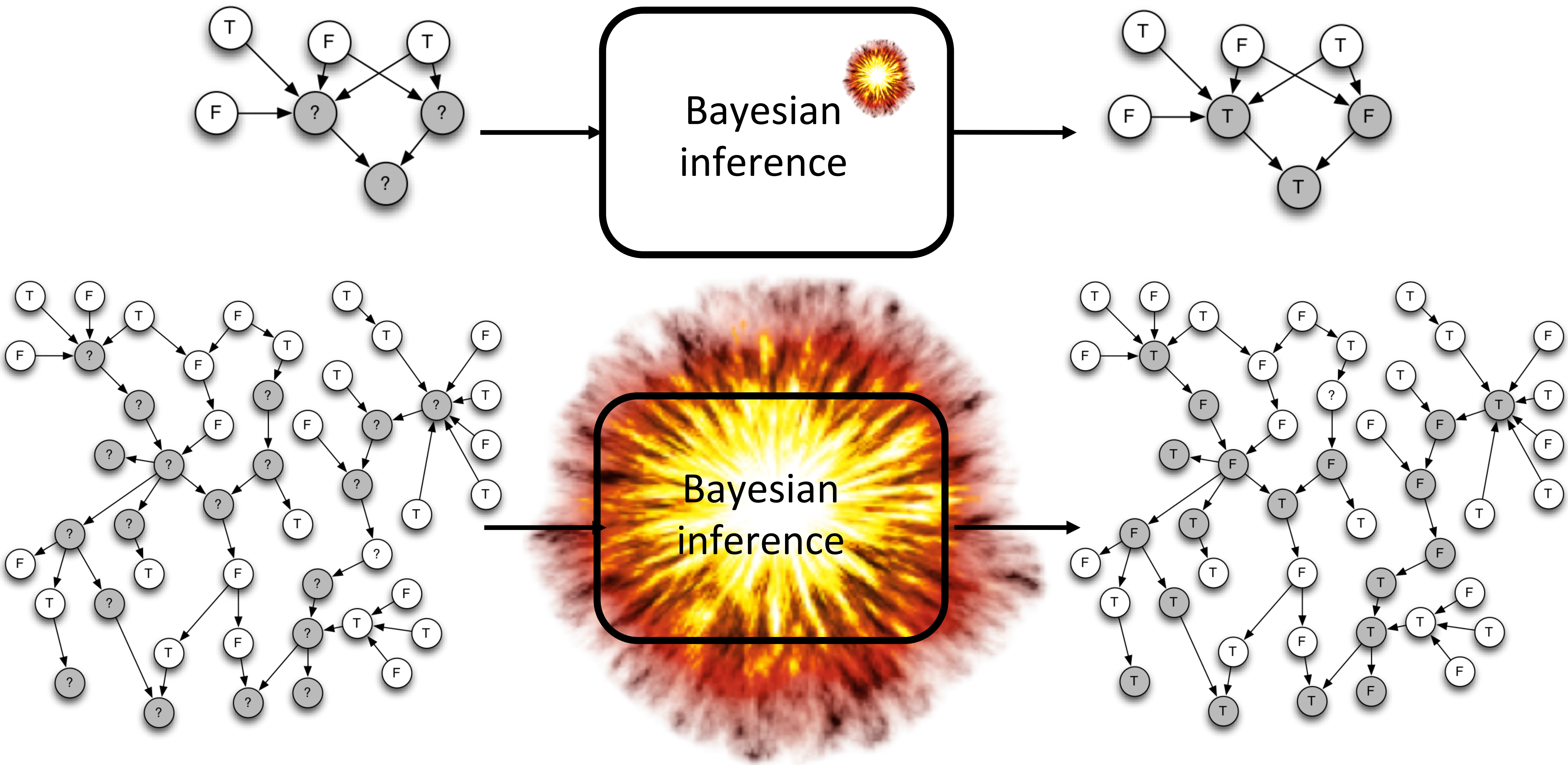

# Intractability is Ubiquitous

" ...it is very widely assumed on inductive grounds by those who model cognitive processes that pretty much any interesting computational problem is super-polynomial in the worst case." (Samuels, 2005)

# Table 1. Examples of (purportedly) intractable computational-level theories

| Cognitive domain | Cognitive model | References |
|---|---|---|
| Analogy | Structure-mapping theory | Gentner (1985) |
| Belief Fixation | Maximum aposterior probability | Abdelbar & Hedetniemi (1998) |
| Belief Fixation | Constraint satisfaction | Thagard (2000) |
| Belief Revision | Default logic | Reiter (1980) |
| Belief Revision | Bayesian belief updating | Cooper (1990) |
| Categorization | Simplicity model | Pothos & Chater (2001, 2002) |
| Decision-making | Bayesian decision-making | Dayan & Daw (2008) |
| Decision-making | Subset choice | van Rooij et al. (2005) |
| Language | Grammar learning | Ristad (1990) |
| Network learning | Weight assignment | Judd (1990) |
| Network settling | Harmony maximization | Bruck & Goodman (1990) |
| Planning | STRIPS | Bylander (1994) |
| Similarity | Representational Distortion | Hahn et al. (2005) |
| Vision | Structural information theory | van der Helm (2004) |
| Vision | Bottom-up visual matching | Tsotsos (1991) |

# How have cognitive scientists (not) been dealing with intractability?

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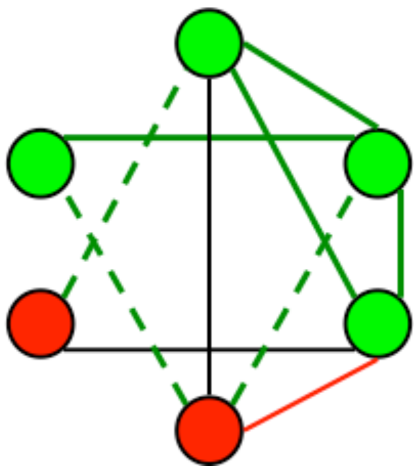
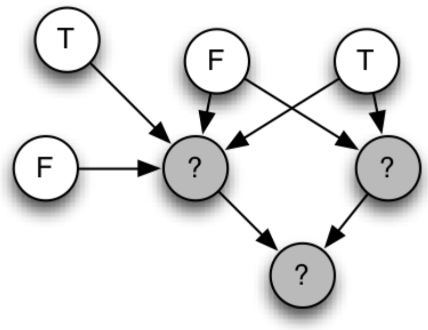
- "Figure pointing"
- Framework rejection

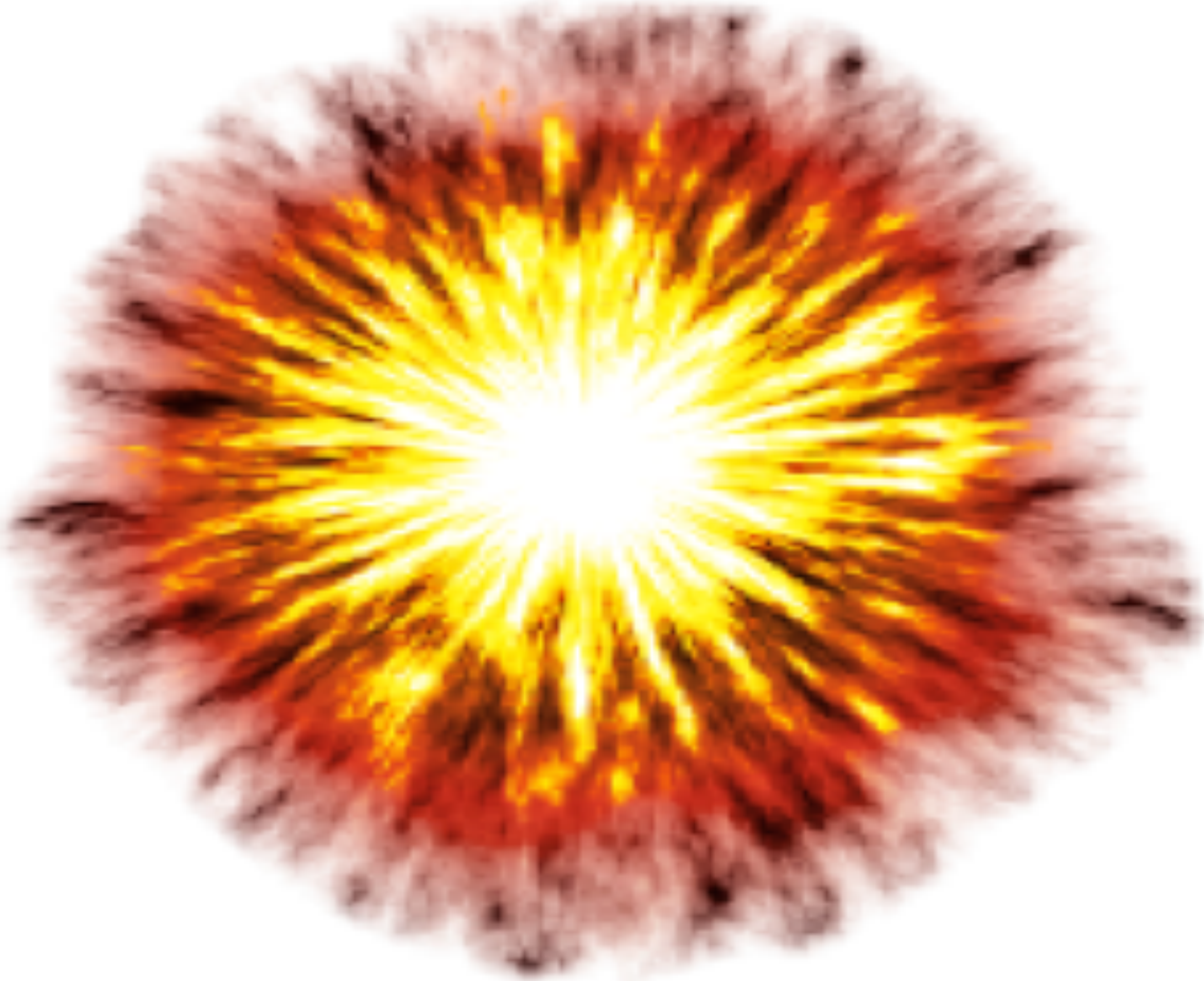
van Rooij (2008) *Cognitive Science*

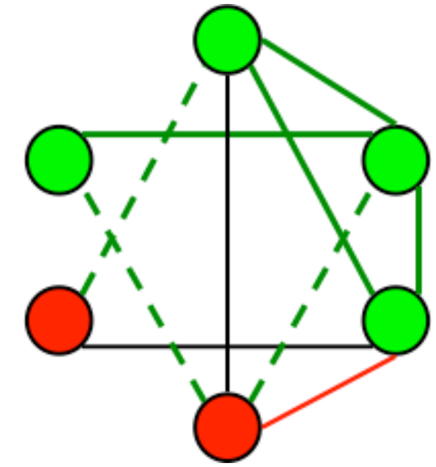
van Rooij (2015) *Proceedings of CogSci2015*.

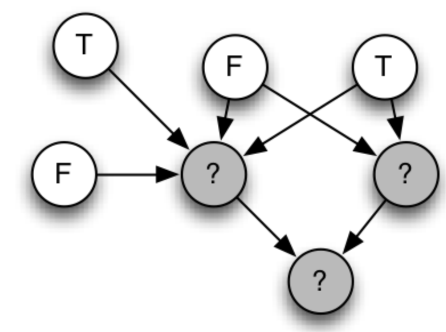
S such that
(1) S & T is consistent,
(2) S & T implies M.







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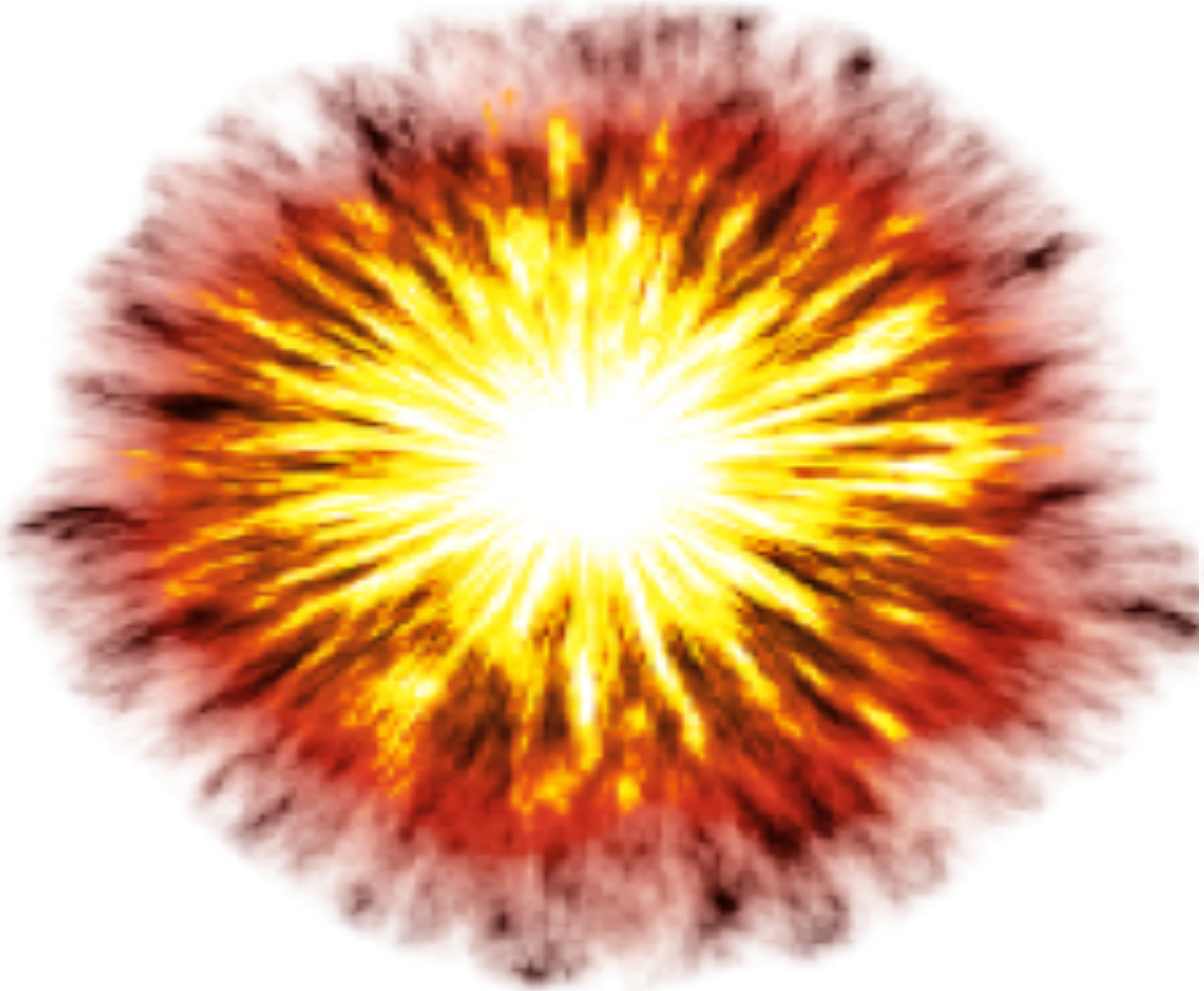
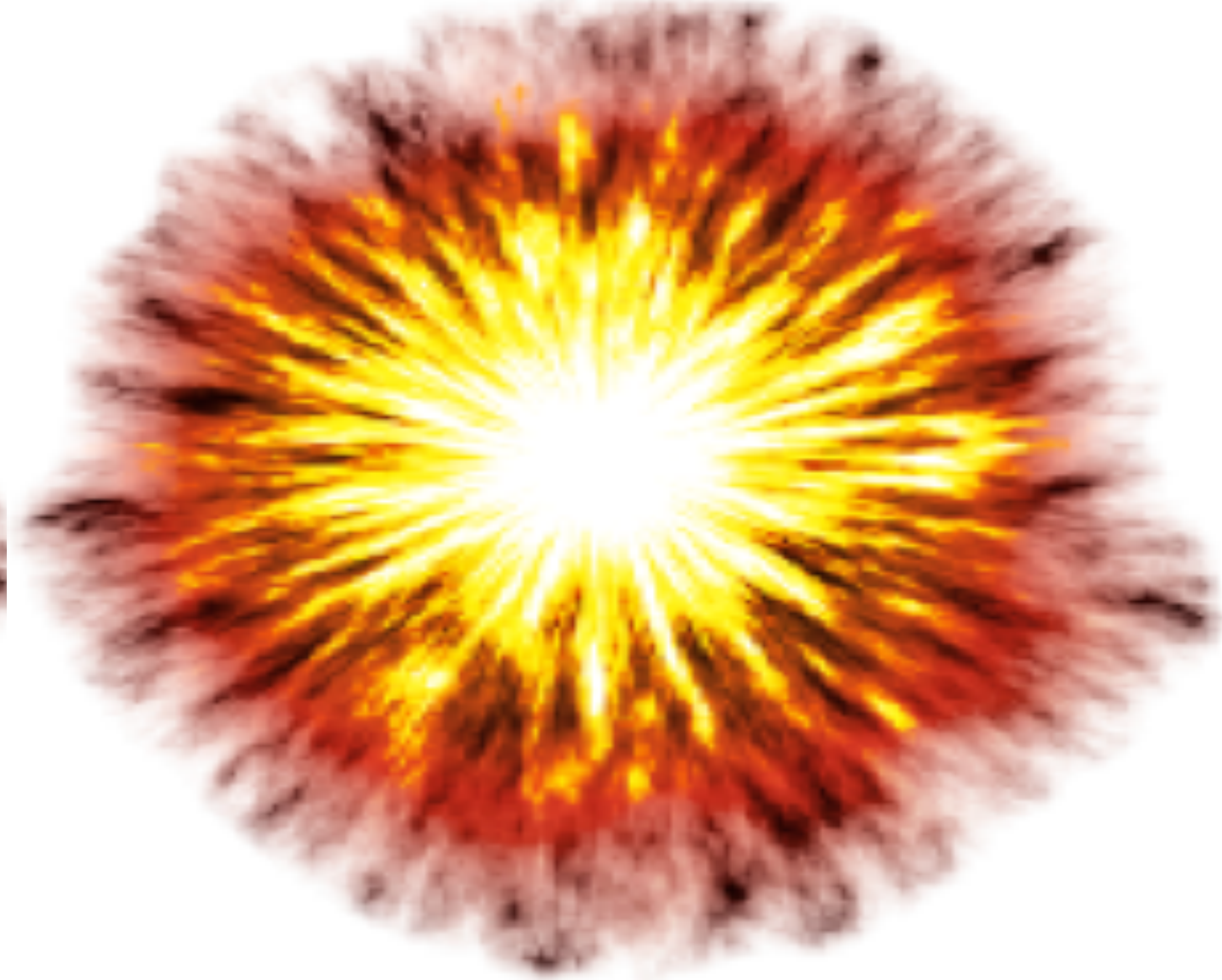
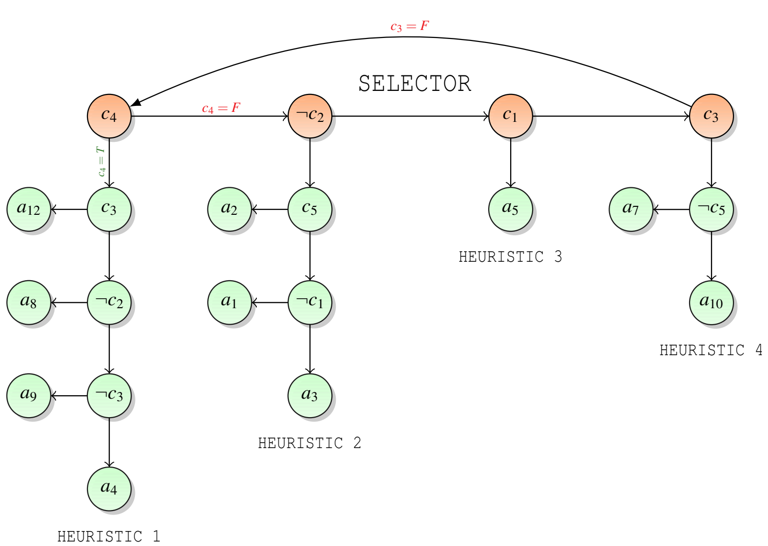
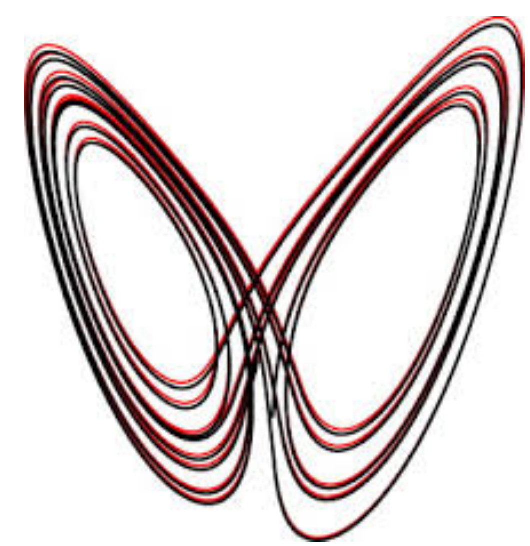






SELECTOR
$c_5 = F$
$c_4 = F$
$c_4 = T$
$c_4$
$\neg c_2$
$c_1$
$c_3$
$a_{12}$
$c_3$
$a_2$
$c_5$
$a_5$
$a_7$
$\neg c_5$
$a_8$
$\neg c_2$
$a_1$
$\neg c_1$
$a_{10}$
$a_9$
$\neg c_3$
$a_3$
$a_4$
HEURISTIC 1
HEURISTIC 2
HEURISTIC 3
HEURISTIC 4

van Rooij, I. (2008).
*Cognitive Science*.
van Rooij, I. (2012).
*Topics in cognitive science*.
Kwisthout, J., et al. (2011).
*Cognitive Science*.
van Rooij et al. (2012, 2014).
*Synthese*.
Otworowska et al. (2017)
*Cognitive Science*.

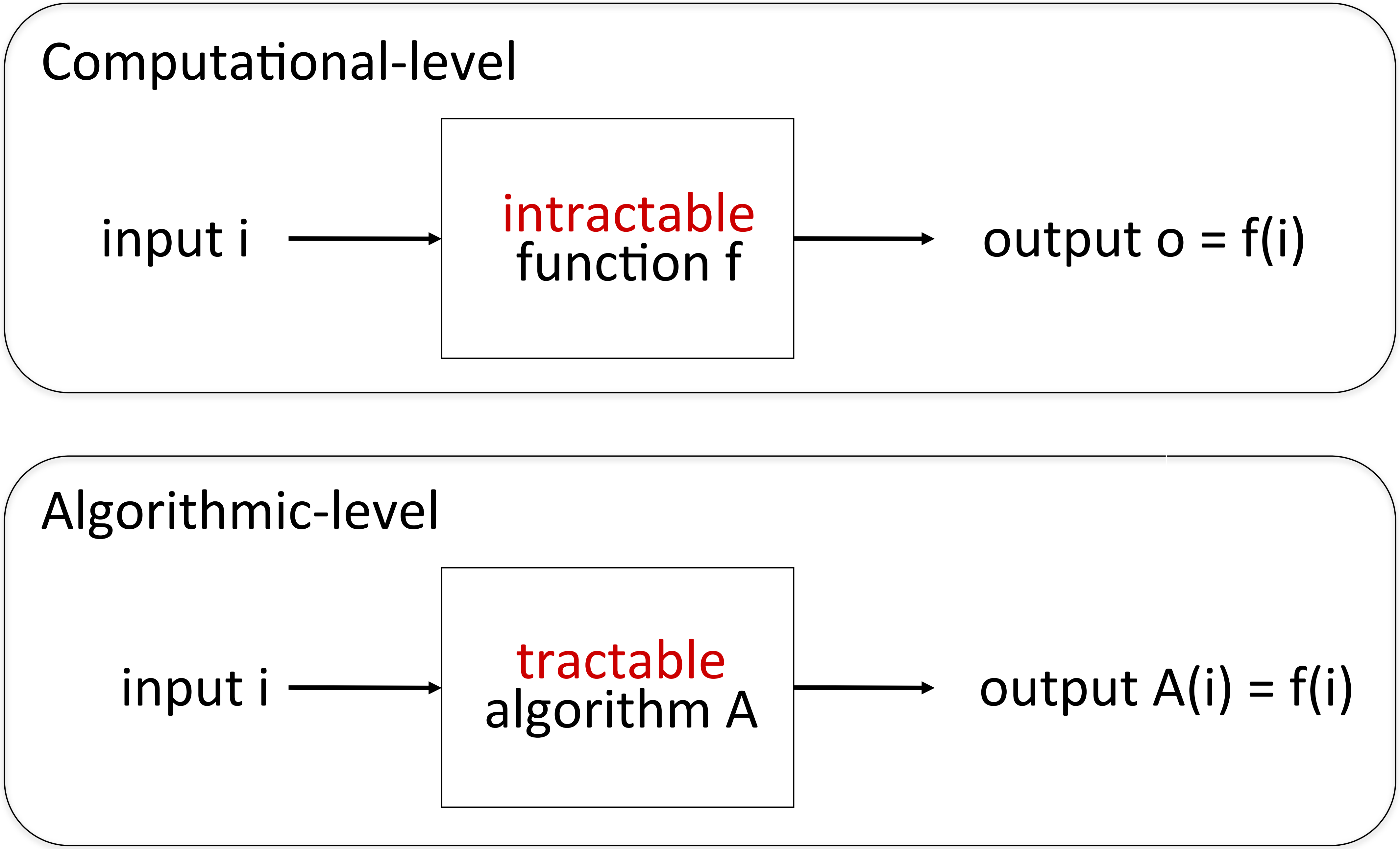
# How have cognitive scientists have (not) been dealing with intractability?

- Average-case Objection
- Super-human Objection
- Parallelism Objection
- Quantum Computing Objection
- **Heuristics Objection**
- **Approximation Objection**
- etc. etc. etc.

van Rooij (2008) *Cognitive Science*

van Rooij et al. (2012) *Synthese*.

# Heuristics as a Coping Strategy



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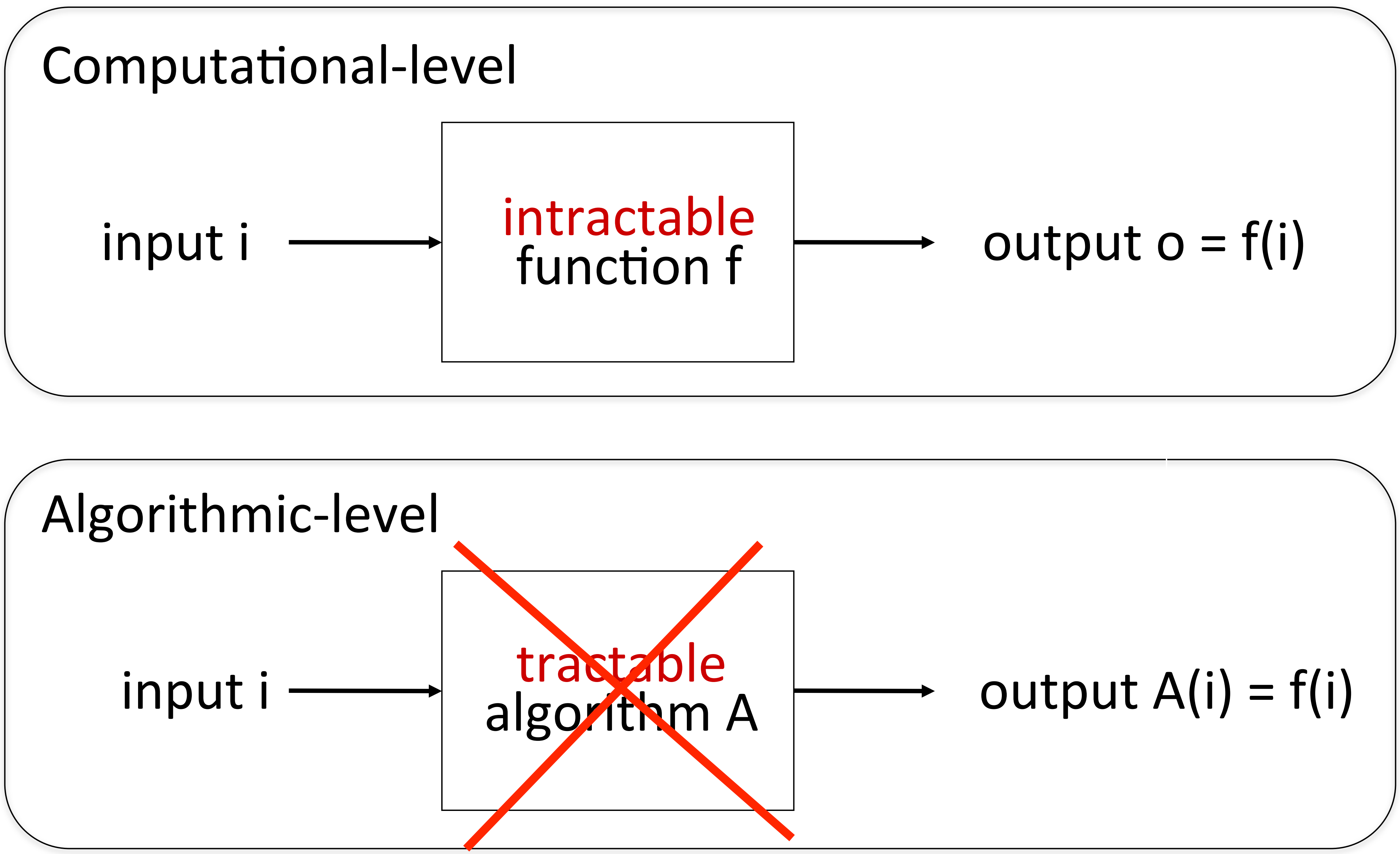
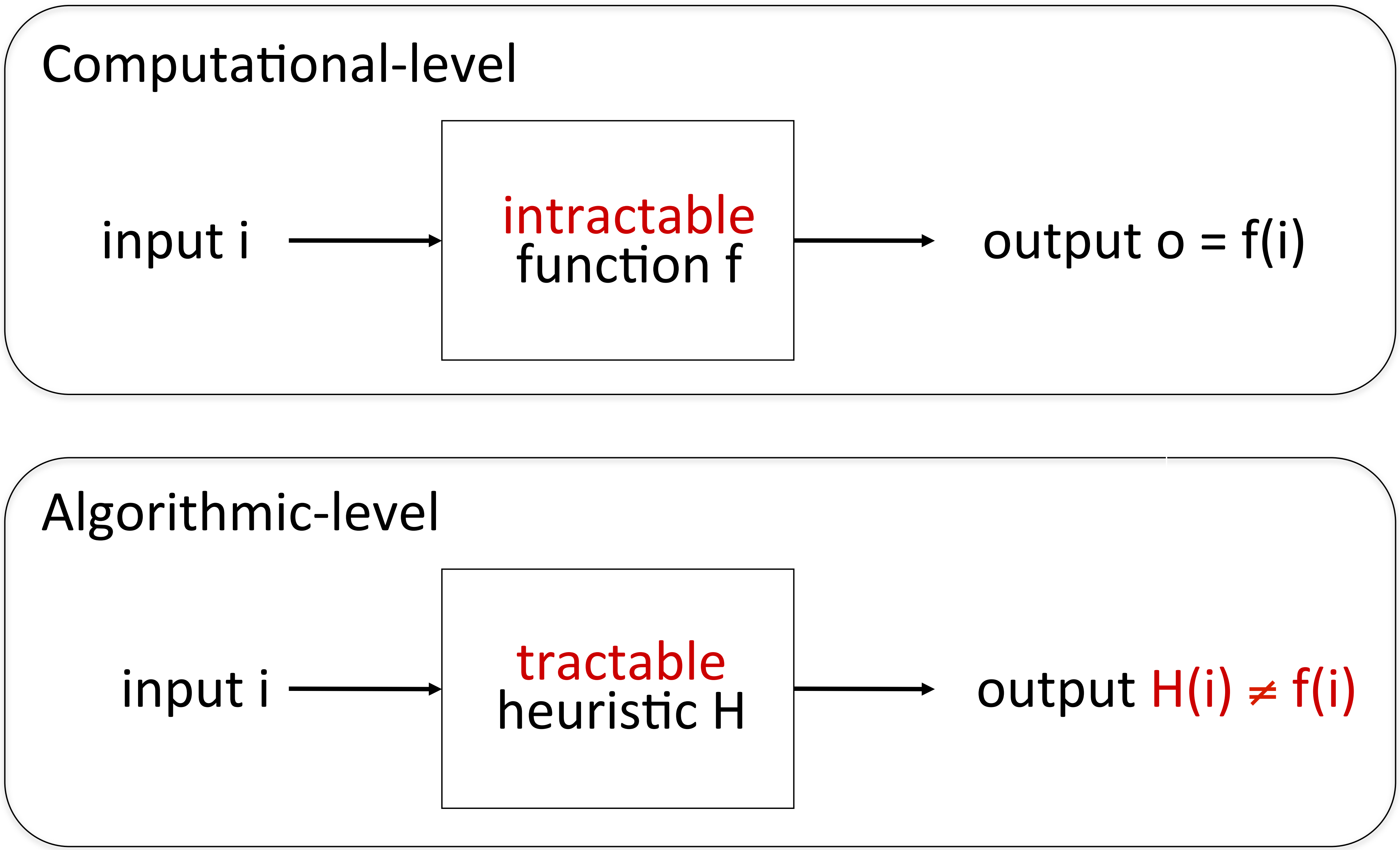
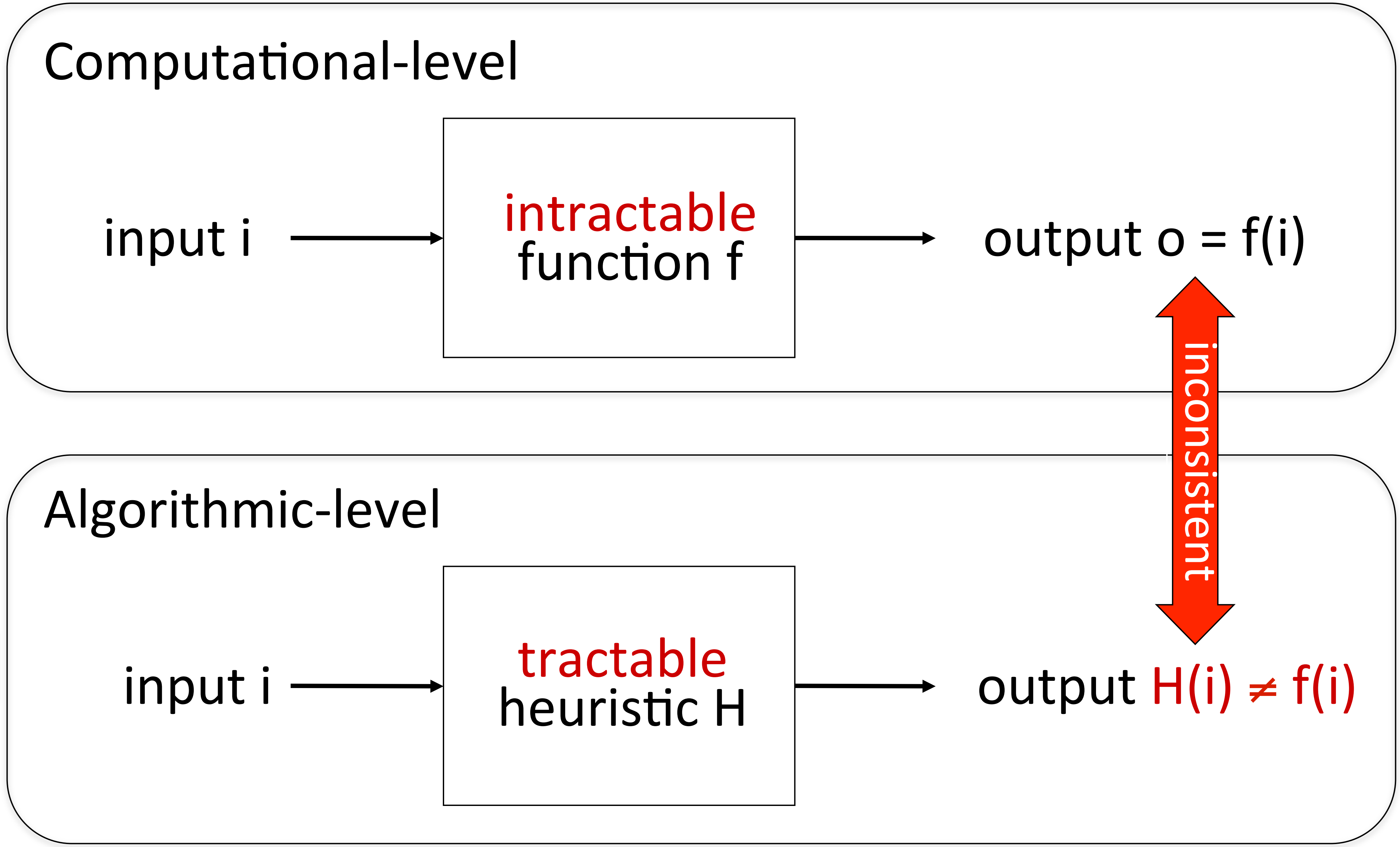


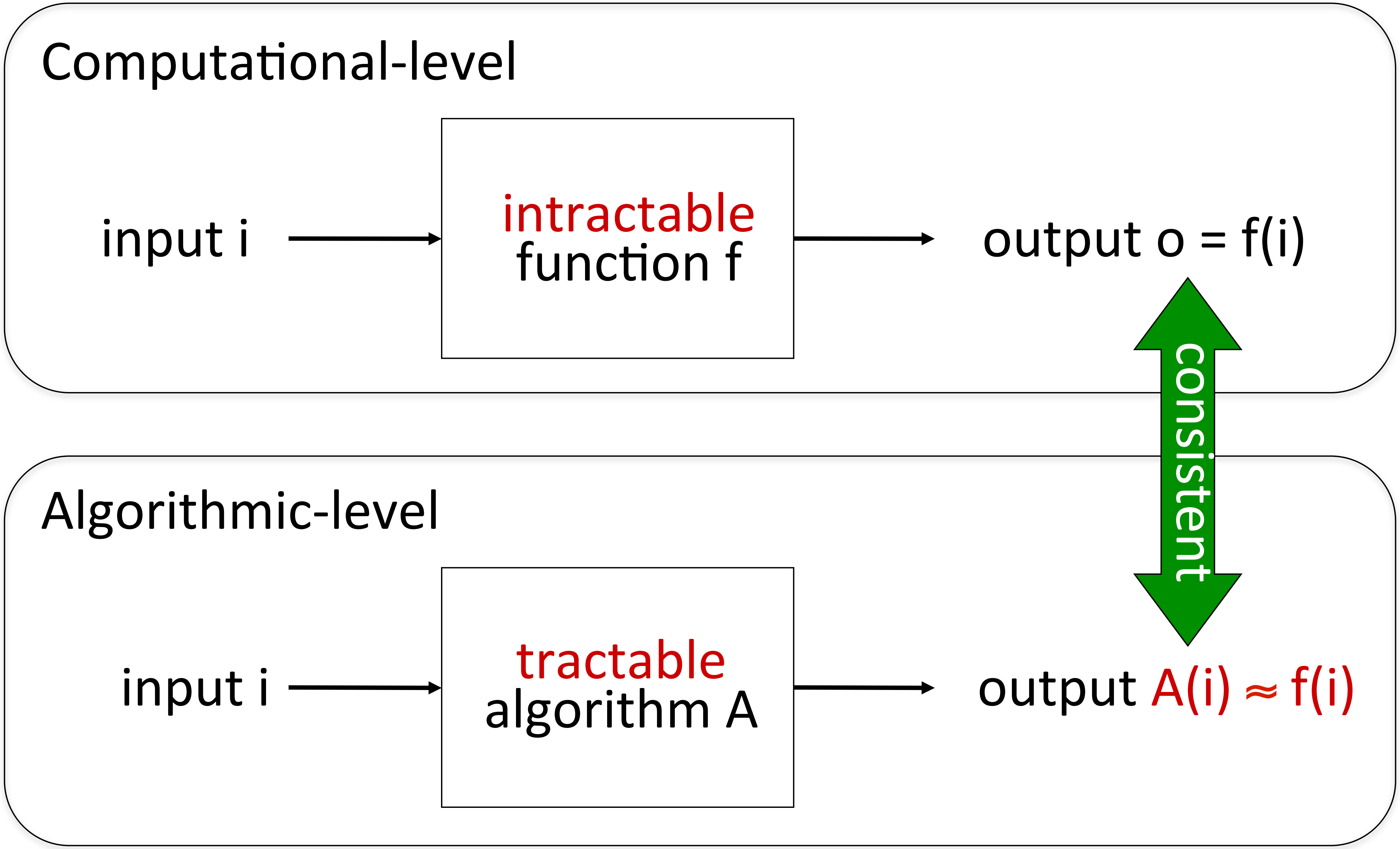
# Heuristics as a Coping Strategy



# Heuristics: The Wrong Way of Coping



# Approximation as a Coping Strategy



# Approximation is often also intractable

Approximating Bayesian inference is intractable, e.g., the following senses:

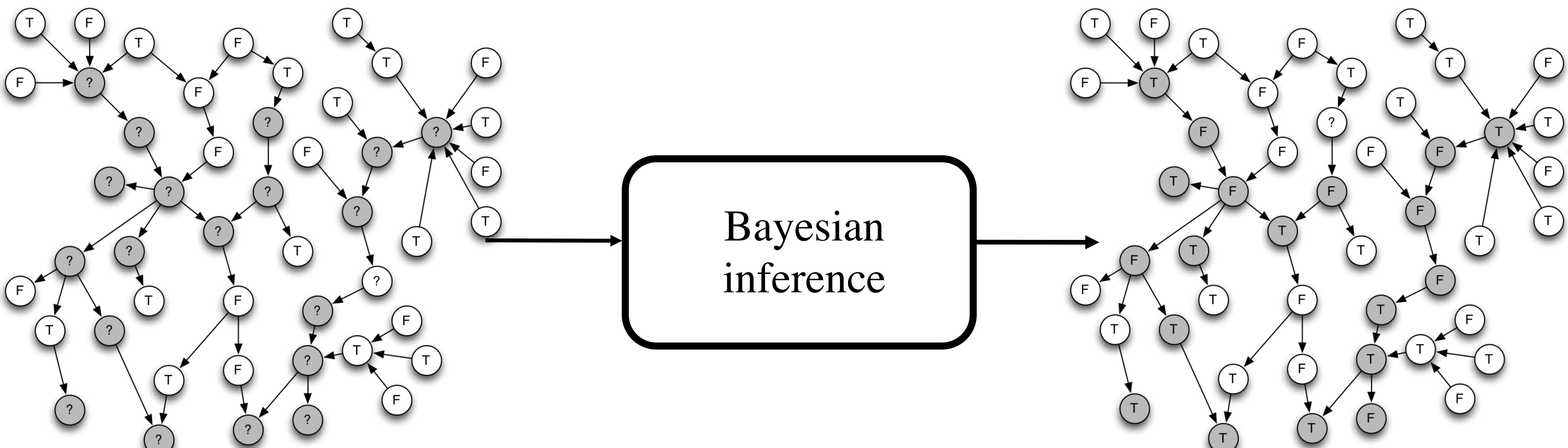
- Computing a truth assignment that has close to maximal probability is NP-hard (Kwisthout, 2011)
- Computing a truth assignment with a posterior probability of at least $q$ for any value $0 < q < 1$ is NP-hard. (Kwisthout, 2011)
- Computing a truth assignment that resembles the most probable truth assignment is NP-hard (Kwisthout, 2014)

# How could cognitive scientists be dealing with intractability?

# The parameterized approach to dealing with intractability

Step 1. Identify parameters of the model that are sources of intractability.

$$\textbf{exp}(n) \longrightarrow \textbf{exp}(k_1, k_2, \ldots k_m)\textbf{poly}(n)$$




**What parameters can you think of?**

# The parameterized approach to dealing with intractability

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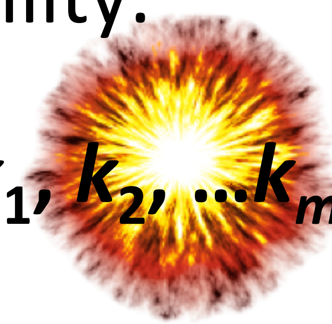
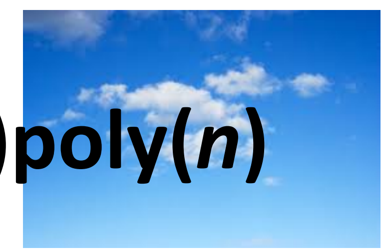
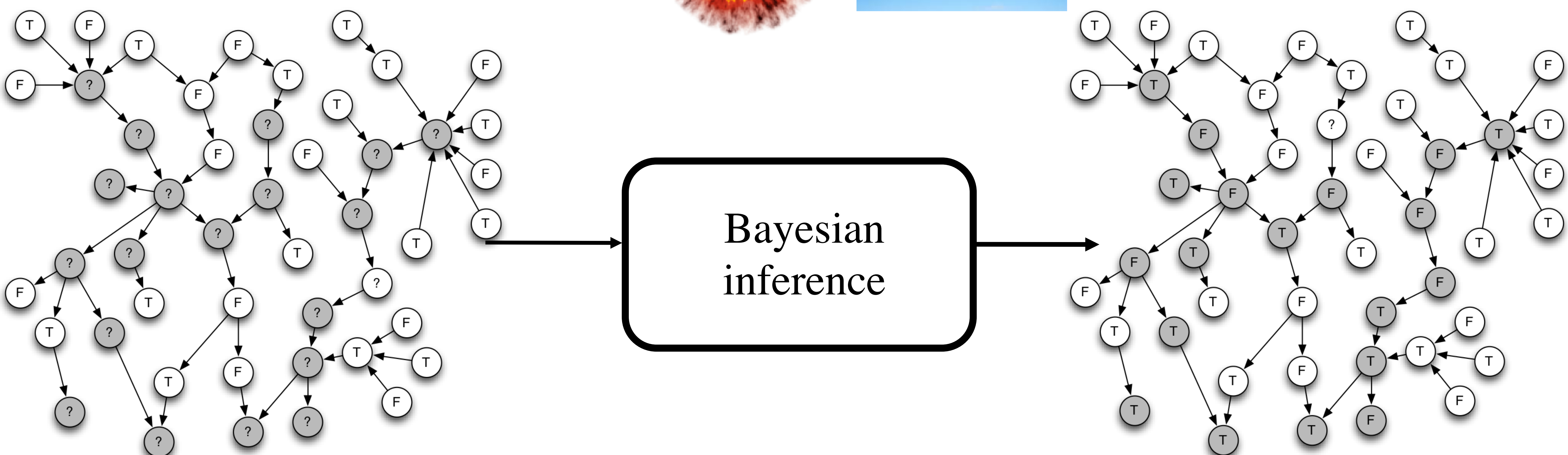



$$\exp(n) \longrightarrow \exp(k_1, k_2, \ldots k_m)\mathrm{poly}(n)$$

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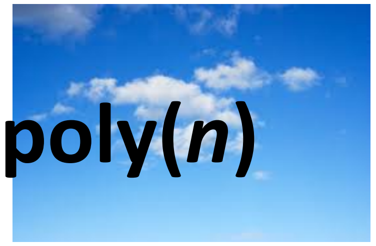

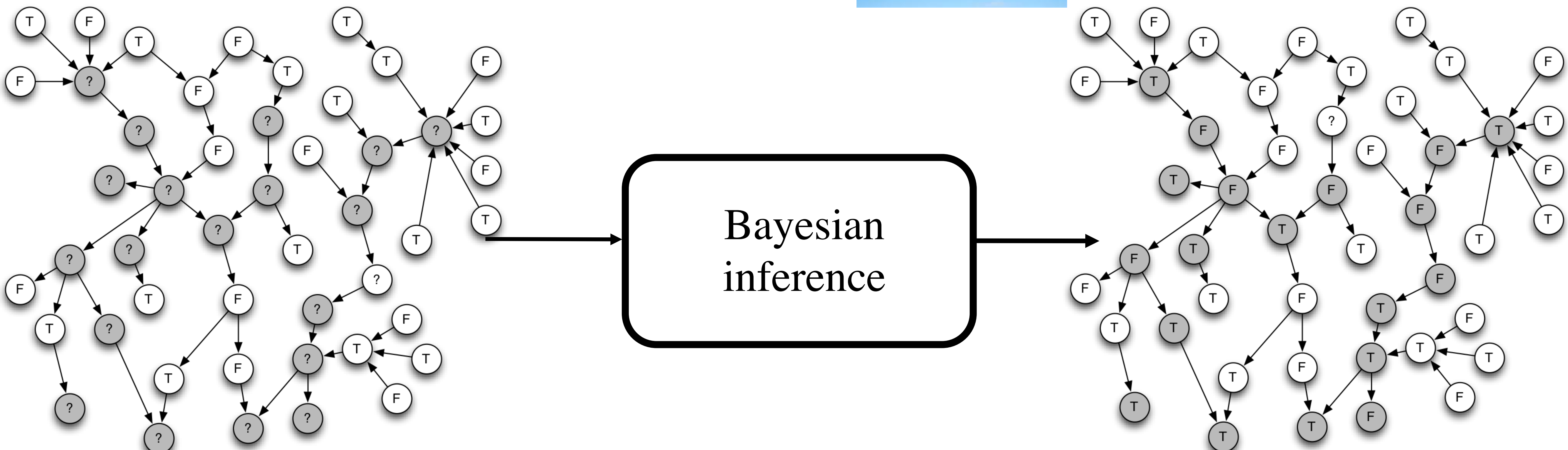


**What parameters can you think of?**

# The parameterized approach to dealing with intractability

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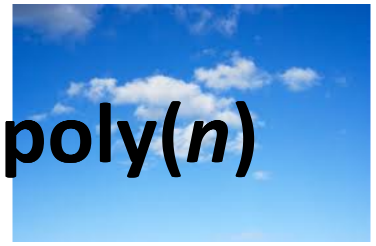
$$\exp(n) \longrightarrow \exp(k_1, k_2, \ldots k_m)\,\mathrm{poly}(n)$$



Step 2. Constrain the model to small values for the parameters $k_1$, $k_2$, …$k_m$. (Note: $n$ can still be large!)

# The parameterized approach to dealing with intractability

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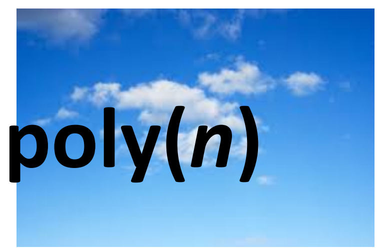
**exp(*n*) ⟶ exp($k_1$, $k_2$, …$k_m$)poly(*n*)**



Step 2. Constrain the model to small values for the parameters $k_1$, $k_2$, …$k_m$. (Note: *n* can still be large!)

Step 3 (validation): Verify that the constraints hold for humans in real-life situations, and test in lab if performance breaks down when parameters are large.

# Conclusions

Cognitive Scientists Should Care about Computational Complexity

**Why?**

1. Intractability prohibits models to scale to the real world.
2. Our intuitions about intractability can be wrong.
3. We need the formal tools from computational complexity theory to verify our intuitions and constrain our models.

# Thank You!

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**Cognition & Intractability**

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with *Mark Blokpoel, Johan Kwisthout & Todd Wareham*